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Research Paper

A note on the domination entropy of graphs

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Abstract. A dominating set of a graph G is a subset D of vertices such that every vertex outside D has a neighbor in D. The domination number of G, denoted by $\gamma(G)$, is the minimum cardinality amongst all dominating sets of G. The domination entropy of G, denoted by $I_{dom}(G)$ is defined as $I_{dom}(G) = -\sum_{i=1}^{k} \frac{d_i(G)}{\gamma_S(G)} \log(\frac{d_i(G)}{\gamma_S(G)})$, where $\gamma_S(G)$ is the number of all dominating sets of G and $d_i(G)$ is the number of dominating sets of cardinality i. A graph G is C_4-free if it does not contain a 4-cycle as a subgraph. In this note we first determine the domination entropy in the graphs whose complements are C_4-free. We then propose an algorithm that computes the domination entropy in any given graph. We also consider circulant graphs G and determine $d_i(G)$ under certain conditions on i.

Keywords. information, domination polynomial, domination entropy, algorithm, circulant graph.
Mathematics Subject Classification (2020): 05C69.

1 Introduction

Let $G = (V, E)$ be a simple graph with vertex set V and edge set E. The *order* of G is $|V|$ and the *size* of G is $|E|$. The *open neighborhood* of a vertex v in a graph G is the set of all vertices adjacent to v, and is denoted by $N(v)$ or $N_G(v)$ to refer it to v. The *degree* of v is $\deg(v) = |N(v)|$. The open neighborhood of a vertex set S is $N(S) = \cup_{v \in S} N(v)$. A graph

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G is C_4-free if it does not contain a C_4 as a subgraph. The *girth* of a graph is the length of a shortest cycle. A *dominating set* of a graph G is a subset D of vertices such that every vertex outside D has a neighbor in D. The *domination number* of G, denoted by $\gamma(G)$, is the minimum cardinality amongst all dominating sets of G. For a graph G of order n the domination polynomial of G, denoted by $D(G,x)$, is defined as follows

$$D(G,x) = \sum_{j=\gamma(G)}^{n} d_j(G)x^j,$$

where, $d_j(G)$ is the number of all dominating sets on the graph G of cardinality j. The domination polynomials where obtained for very few classes of graphs, including complete graphs, complete bipartite graph, paths [2], cycles [3], friendship graphs [4], and caterpillar graphs [17], is still unknown in many classes of graphs.

The concept of information entropy (also known as Shannon entropy) was introduced in 1948 by Shannon [15]. Shannon entropy defines a data communication system composed of a source of data, a communication channel, and a receiver, such that the receiver can be able to identify what data was generated by the source, based on the signal it receives through the channel. Various types of entropy have been already considered, see for example, [7, 9, 13, 16]. It was considered in graphs by Rashevsky in 1955 [14] by considering vertex degrees of graphs. This concept was further studied, for example, in [7, 8, 10, 19].

Dehmer in 2008 [9] studied information processing in complex networks by considering graph entropy and information functionals as $I(G) = -\sum_{i=1}^{n} p_i \log p_i$, where the p_is are vertex probabilities and the logarithmic phrases have base 2. Recently, Sahin [16] considered a new information functional and introduced the domination entropy of graphs. For a graph G of order n without an isolated vertex, the information functional is $p_i = \frac{d_i(G)}{\sum_{j=1}^{n} d_j(G)}$, where $d_i(G)$ is the number of dominating sets of G of cardinality i. The *domination entropy* of G, denoted by $I_{dom}(G)$ is as follows:

$$I_{dom}(G) = \log(\gamma_s(G)) - \frac{1}{\gamma_s(G)} \sum_{i=\gamma(G)}^{n-2} d_i(G) \log(d_i(G)) - \frac{n \log n}{\gamma_s(G)}, \quad (1)$$

where $\gamma_s(G)$ is the number of all dominating sets of G. Sahin [16] determined the domination entropy in some families of graphs including complete graphs, star graphs, double-star graphs, comb graph and friendship graphs, based on the known results on domination polynomials of these graphs. In this paper we consider domination entropy in graphs whose complement are C_4-free, as important classes of graphs in the information theory and coding. We note that much have been written on graphs with high girth in information theory and coding, see for example, [5, 6, 11, 12].

The organization of the paper is as follows. In Section 2 we first determine the domination entropy in graphs whose complement are C_4-free, and then we present an algorithm namely Algorithm 2.2 that enables us to compute the domination entropy of any given graph G. In Section 3 we focus on a famous family of graph namely circulant graphs. We first determine several domination polynomial coefficients in the general, and then present a new algorithm namely Algorithm 3.1 which leads to a conjecture on the coefficients of the domination polynomials of circulant graphs under certain conditions.

2 Complements of C_4-free graphs

We first determine the domination entropy in graphs whose complement are C_4-free.

Theorem 2.1. *Let G be a graph with vertex set $\{v_1, ..., v_n\}$ such that $\overline{G}$ is C_4-free, and let d_j be the number of dominating sets of G of cardinality j. Then:*

(I) $d_j = 0$ if $j < \gamma(G)$, and

$$d_j = \binom{n}{j} - \sum_{v_i:\ n-1-\deg(v_i) \geq j} \binom{n-1-\deg(v_i)}{j}$$

if $j \geq \gamma(G)$.

(II)

$$I_{dom}(G) = \log(\gamma_s) - \frac{1}{\gamma_s} \sum_{j=\gamma(G)}^{n-2} d_j \log(d_j) - \frac{n \log n}{\gamma_s(G)},$$

where $\gamma_s = \sum_{j=\gamma(G)}^{n} \left(\binom{n}{j} - \sum_{v_i:\ n-1-\deg(v_i) \geq j} \binom{n-1-\deg(v_i)}{j} \right)$ and d_j is described in (I).

Proof. (I) The proof is obvious for $j < \gamma(G)$, thus assume that $j \geq \gamma(G)$. Let A_j be set of all j-subsets of $V(G)$ that are not dominating sets of G. Then clearly

$$d_j = \binom{n}{j} - |A_j|. \tag{2}$$

For each set $S \in A_j$, clearly there is a vertex v_i in G that is not dominated by S, and so v_i is adjacent to all vertices of S in $\overline{G}$, that is, $v_i \in \bigcap_{s \in S} N_{\overline{G}}(s)$. Since $\overline{G}$ is C_4-free, we find that $\bigcap_{s \in S} N_{\overline{G}}(s) = \{v_i\}$. Then $S \subseteq N_{\overline{G}}(v_i)$, that is, S is a j-subset of $N_{\overline{G}}(v_i)$, where $N_{\overline{G}}(v_i) \geq j$. Since $N_{\overline{G}}(v_i) = \deg_{\overline{G}}(v_i) = n - 1 - \deg(v_i)$, the proof of (I) is complete.

(II) By (1),

$$I_{dom}(G) = \log(\gamma_s(G)) - \frac{1}{\gamma_s(G)} \sum_{j=\gamma(G)}^{n-2} d_j(G) \log(d_j(G)) - \frac{n \log n}{\gamma_s(G)},$$

where $\gamma_s(G)$ is the number of all dominating sets of G. Clearly $\gamma_s(G)$ is the number of all dominating sets of G of all cardinalities j, where $\gamma(G) \leq j \leq n$. Now replacing all such d_js ($j \geq \gamma(G)$) with that stated in (I) yields the desired result. □

Following the proof of Theorem 2.1, $d_j = \binom{n}{j} - |A_j|$, where A_j is set of all j-subsets of $V(G)$ that are not dominating sets of G. Clearly $|A_j|$ is the number of j-subsets of $\overline{G}$ that have at least a common neighbor in $\overline{G}$. In this section we propose an algorithm, namely, Algorithm 3.2 to compute $|A_j|$ in any graph G, thus enabling to compute the domination entropy. For this purpose we first give an algorithm, namely, Algorithm, 3.1 which computes the complement of a graph.

Algorithm 1 Compute-Complement graph(G)

Input: A graph G of order n with $V(G) = \{0,1,\cdots,n-1\}$
Output: The complement graph of G

$num\_vertices \leftarrow n$ $complement \leftarrow [\ ]$ (an empty list of size $num\_vertices$) **for** $i = 0,\cdots,num\_vertices - 1$ **do**
  for $j = 0,\cdots,num\_vertices - 1$ **do**
    if $i \neq j$ *and* $j \notin N_G(i)$ **then**
      append j to $complement[i]$
    end
  end
end
Return $\overline{G}$

Algorithm 2 Compute $|A_j|$

Input: A graph G of order n with vertex set $V(G) = \{0,1,\cdots,n-1\}$ and an integer $j \leq n$
Output: $|A_j|$

if $j < \gamma(G)$ **then**
  $|A_j| = \binom{n}{j}$
end
else
  Calculate Compute-Complement graph(G) $complement \leftarrow \overline{G}$ $|A_j| \leftarrow 0$
  for *each j-vertex combination in* $\{0,1,\cdots,n-1\}$ **do**
    $k_j \leftarrow$ first element of the j-vertex combination $common\_neighbors \leftarrow$ set($complement[k_j]$) **for** *each vertex k' in the j-vertex combination after the first element* **do**
      $common\_neighbors \leftarrow common\_neighbors \cap$ set($complement[k']$)
    end
    if $|common\_neighbors| > 0$ **then**
      $|A_j| \leftarrow |A_j| + 1$
    end
  end
end
Return $|A_j|$

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3 Circulant graphs

In this section we consider a famous family of graphs, namely, circulant graphs. The circulant graph $C_n(1,2,...,k)$ is a graph with vertex set $V = \{v_1,...,v_n\}$ such that for each i, the vertex v_i is adjacent to $v_{i+1},...,v_{i+k}$, where the addition is in modulo n. Figure 1 depicted the circulant graph $C_{12}(1,2,3)$.

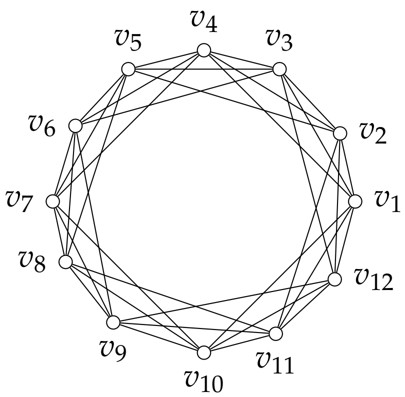

Figure 1. Graph $C_{12}(1,2,3)$.

Note that $C_n(1,2,...,k)$ is a regular graph. Furthermore if $n \geq 2k+1$ then it is $2k$-regular, thus its complement is $n-2k-1$-regular. Thus we have the following result.

Theorem 3.1. *Let $G = C_n(1,2,...,k)$ be a circulant graph with $n \geq 2k+1$. Then*
(I) For $n-2k \leq j \leq n$, $d_j = \binom{n}{j}$,
(II) For $j = n-2k-1$, $d_j = \binom{n}{j} - n$,
(III) For $j = n-2k-2$, $d_j = \binom{n}{j} - nj$.

Proof. From Theorem 2.1, we have $d_j = \binom{n}{j} - |A_j|$. For $n-2k \leq j \leq n$, clearly, $|A_j| = 0$. Thus (I) follows. We next prove (II). Assume now that $j = n-2k-1$. Let S be a j-subset of G such that all vertices in S have a common neighbor in $\overline{G}$. Since $\overline{G}$ is $n-2k-1$-regular, we have $S = N_{\overline{G}}(v_i)$ for some $i \in \{1,2,...,n\}$. Then v_i is the only vertex in $\overline{G}$ that is adjacent to S is $\overline{G}$. On the other hand for each integer $i = 1,...,n$, $S = N_{\overline{G}}(v_i) \in A_j$. We deduce that $A_j = \{N_{\overline{G}}(v_1), N_{\overline{G}}(v_2),...,N_{\overline{G}}(v_n)\}$. Consequently, $|A_j| = n$, and thus the result follows.

(III) Assume that $j = n-2k-2$. Let S be a j-subset of G such that all vertices in S have a common neighbor in $\overline{G}$. Then $S \subseteq N_{\overline{G}}(v_i)$ for some $i \in \{1,2,...,n\}$. It is evident that $S \nsubseteq N_{\overline{G}}(v_l)$ for $l \neq i$. On the other hand for each integer $i = 1,...,n$, there are $\binom{n-2k-1}{n-2k-2} = j$ set S with $|S| = j$ and $S \subseteq N_{\overline{G}}(v_i)$. We deduce that $A_j = nj$ and the result follows. □

For $j \leq n-2k-3$, computing d_j as a formula is complicated, and the only option is applied Algorithm 2 when n is small enough. We applied Algorithm 2 on circulant graphs $C_n(1,2)$ and $C_n(1,2,3)$ with $n \leq 18$, and using (3) we obtained the Tables 1 and 2 (see Appendix A.). According to the values of d_j for $j \leq n-2k-3$ one can have the following new point of view which results in a conjecture on d_j under some certain conditions. As it was seen, for each integer j, A_j is the set of all j-subsets of $V(G)$ that are not dominating sets of G. Then A_j is the set of all j-subsets of $V(G)$ that have at least one common neighbors in $\overline{G}$. We can write $|A_j| = \sum_{i=1}^{n-1} M_i$, where M_i is the number of j-subsets of $V(G)$ with i common neighbors in $\overline{G}$. Let M be $(n-\gamma(G)) \times (n-1)$ matrix whose rows are indexed with $\gamma, \gamma+1, ..., n-1$ and whose columns are indexed by M_1, M_2, ..., M_{n-1}, and the ij entry of M is M_j, where $|A|_i = \sum_{j=1}^{n-1} M_j$. The following algorithm 4.1 can be applied on $C_n(1,2,...,k)$ for all k and n to compute the matrix M. Note that it can be seen that $\gamma(C_n(1,2,...,k)) = \lceil \frac{n}{2k+1} \rceil$.

Algorithm 3 Compute Matrix M

Input: The circulant graph $C_n(1,2,\cdots,k)$
Output: Matrix M

$max \leftarrow n-(2k+1)$ $min \leftarrow \lceil \frac{n}{2k+1} \rceil$ Calculate Compute-Complement graph($C_n(1,2,\cdots,k)$) $complement \leftarrow \overline{C_n(1,2,\cdots,k)}$ Initialize a $(n-\min)\times(n-1)$ matrix M with all zero entries
for $\min \leq i < \max + 1$ **do**
  for *each i-vertex combinations in* $\{0,1,...,n-1\}$ **do**
    $k_i \leftarrow$ first element of the i-vertex combination $common\_neighbors \leftarrow$ set($complement[k_i]$) **for** *each vertex k' in the i-vertex combination after the first element* **do**
      $common\_neighbors \leftarrow common\_neighbors \cap$ set($complement[k']$)
    end
    if $|common\_neighbors| > 0$ **then**
      for $j = 0,\cdots,|common\_neighbors|-1$ **do**
        if $|common\_neighbors| = j+1$ **then**
          $M[i-\min, j] \leftarrow M[i-\min, j] + 1$
        end
      end
    end
  end
end
Return M

Applying Algorithm 3 on the circulant graphs of small orders yeilds the following conjecture.

Conjecture 3.2. *Let $G = C_n(1,2,...,k)$ be a circulant graph with $n \leq 4k+1-\lceil \frac{n}{2k+1} \rceil$ and $\lceil \frac{n}{2k+1} \rceil \leq j \leq n-2k-3$. Then $d_j = \binom{n}{j} - n\sum_{i=1}^{n-2k-j} \binom{i+j-3}{i-1}$.*

An example of applying Algorithm 3 on the circulant graph $C_{16}(1,2,3)$ posed in Table 3 (Appendix B.) which confirms the validity of Conjecture 1. Note that for each integer $j \geq 3 = \gamma(C_{16}(1,2,3))$, $d_j = \binom{n}{j} - t_j$, where $t_j = |A_j|$ is the number of j-subset S of G such that all vertices in S have a common neighbor in $\overline{G}$. We can write $t_j = M_1 + M_2 + ... + M_{n-1}$, where for each i, M_i is the number of j-subset S of G such that all vertices in S have precisely i common neighbor in $\overline{G}$.

4 Conclusion

In this paper we studied the domination entropy in graphs. We determined the domination entropy in graphs whose complements are C_4-free, and proposed an algorithm to compute the domination entropy in any given graph G. We also studied circulant graphs G and determine $d_i(G)$ under certain conditions on i which resulted in a conjecture, namely, Conjecture 3.2. It is a good problem to study these problems for other domination variants.

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Data Availability

Data sharing is not applicable to this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this article.

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Appendix A: Tables 1 and 2.

$n\backslash d_j$	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}
3	3	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	4	6	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	5	10	10	5	1	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	15	20	15	6	1	0	0	0	0	0	0	0	0	0	0	0	0
7	0	14	35	35	21	7	1	0	0	0	0	0	0	0	0	0	0	0
8	0	12	48	70	56	28	8	1	0	0	0	0	0	0	0	0	0	0
9	0	9	57	117	126	84	36	9	1	0	0	0	0	0	0	0	0	0
10	0	5	60	170	242	210	120	45	10	1	0	0	0	0	0	0	0	0
11	0	0	55	220	407	451	330	165	55	11	1	0	0	0	0	0	0	0
12	0	0	40	255	612	852	780	495	220	66	12	1	0	0	0	0	0	0
13	0	0	26	260	832	1443	1625	1274	715	286	78	13	1	0	0	0	0	0
14	0	0	14	238	1022	2219	3040	2891	1988	1001	364	91	14	1	0	0	0	0
15	0	0	5	195	1143	3115	5175	5895	4870	2988	1365	455	105	15	1	0	0	0
16	0	0	0	140	1168	4016	8080	10950	10720	7848	4352	1820	560	120	16	1	0	0
17	0	0	0	85	1088	4777	11645	18700	21505	18513	12189	6171	2380	680	136	17	1	0
18	0	0	0	45	918	5253	15570	30565	39710	39798	30636	18348	8550	3060	816	153	18	1

Table 1. d_j's in $C_n(1,2)$ for $n \leq 18$.

$n\backslash d_j$	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}
3	3	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	4	6	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	5	10	10	5	1	0	0	0	0	0	0	0	0	0	0	0	0	0
6	6	15	20	15	6	1	0	0	0	0	0	0	0	0	0	0	0	0
7	7	21	35	35	21	7	1	0	0	0	0	0	0	0	0	0	0	0
8	0	28	56	70	56	28	8	1	0	0	0	0	0	0	0	0	0	0
9	0	27	84	126	126	84	36	9	1	0	0	0	0	0	0	0	0	0
10	0	25	110	210	252	210	120	45	10	1	0	0	0	0	0	0	0	0
11	0	22	132	319	462	462	330	165	55	11	1	0	0	0	0	0	0	0
12	0	18	148	447	780	924	792	495	220	66	12	1	0	0	0	0	0	0
13	0	13	156	585	1222	1703	1716	1287	715	286	78	13	1	0	0	0	0	0
14	0	7	154	721	1792	2919	3418	3003	2002	1001	364	91	14	1	0	0	0	0
15	0	0	140	840	2478	4690	6330	6420	5005	3003	1365	455	105	15	1	0	0	0
16	0	0	112	924	3248	7112	10992	12742	11424	8008	4368	1820	560	120	16	1	0	0
17	0	0	85	952	4046	10234	18020	23698	24157	19431	12376	6188	2380	680	136	17	1	0
18	0	0	60	927	4788	14028	28044	41598	47810	43578	31806	18564	8568	3060	816	153	18	1

Table 2. d_j's in $C_n(1,2,3)$ for $n \leq 18$.

Appendix B: Table 3.

$\|A\|_i\backslash M_j$	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}	M_{13}	M_{14}	M_{15}
$\|A\|_3$	112	96	80	64	48	32	16	0	0	0	0	0	0	0	0
$\|A\|_4$	336	240	160	96	48	16	0	0	0	0	0	0	0	0	0
$\|A\|_5$	560	320	160	64	16	0	0	0	0	0	0	0	0	0	0
$\|A\|_6$	560	240	80	16	0	0	0	0	0	0	0	0	0	0	0
$\|A\|_7$	336	96	16	0	0	0	0	0	0	0	0	0	0	0	0
$\|A\|_8$	112	16	0	0	0	0	0	0	0	0	0	0	0	0	0
$\|A\|_9$	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\|A\|_{10}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\|A\|_{11}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\|A\|_{12}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\|A\|_{13}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\|A\|_{14}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\|A\|_{15}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 3. Determining $M_j(j = 3,...,16)$ in $C_{16}(1,2,3)$. Note that it is easy to see that $|A|_j = n\sum_{i=1}^{n-2k-j}\binom{i+j-3}{i-1}$ for $j = 3,4,...,7$.

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