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Research Paper

On the weighted bond additive indices of some nanostructures

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Abstract. Topological indices are a type of molecular descriptor which serves as a tool for property analysis of chemical compounds based on their molecular structure. The edge Mostar index is a recently introduced topological index, which serves as a measure of peripherality of graphs. Two new weighted versions of edge Mostar index have been proposed recently, namely additively weighted and multiplicatively weighted edge Mostar indices. In this paper, we determine the explicit expressions of different weighted versions of edge Mostar indices of carbon nanostructures such as $T^1UC_4C_8[p,q]$-lattice, $T^2UC_4C_8[p,q]$-lattice using a variant of cut method.

Keywords. Mostar index, additively weighted Mostar index, multiplicatively weighted Mostar index, nanostructures.

Mathematics Subject Classification (2020): $92E10$, $05C92$, $05C09$, $05C35$.

As per the IUPAC [16], a topological index is a numerical quantity which can be associated with physical-chemical properties of molecular compounds. It helps predict the structural property of chemical compounds without being undergone chemical testing. Numerous topological indices are available in literature based on different graph theoretical parameters such as distance, degree, and spectrum. The Mostar index is a recently introduced bond-additive distance-based graph invariant which has been extensively used to study the peripherality [6] of graphs. Mostar index of a graph $G = (V, E)$ is denoted by $Mo(G)$ [9] and

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is defined as

$$Mo(G) = \sum_{e=uv \in E} |n_u(e|G) - n_v(e|G)|,$$

where $n_u(e|G)$ is the number of vertices of G closer to u than v and $n_v(e|G)$ is the number of vertices of G closer to v than u. Several weighted versions of Mostar indices have been proposed recently [8, 15]. Prominent ones were additively weighted Mostar index and multiplicatively weighted Mostar index [6] which are defined as

$$Mo_A(G) = \sum_{e=uv \in E} (d(u) + d(v))|n_u(e|G) - n_v(e|G)|,$$

and

$$Mo_M(G) = \sum_{e=uv \in E} (d(u)d(v))|n_u(e|G) - n_v(e|G)|,$$

respectively. For more works on Mostar indices, see [2, 3, 5–7, 11, 14]. Similar extensions on edge Mostar index was also done recently, which are called additively weighted edge Mostar index [4] and multiplicatively weighted edge Mostar index, they are defined by

$$MoA_e(G) = \sum_{e=uv \in E} (d(u) + d(v))|m_u(e|G) - m_v(e|G)|,$$

and

$$MoM_e(G) = \sum_{e=uv \in E} (d(u)d(v))|m_u(e|G) - m_v(e|G)|,$$

respectively, where $m_u(e|G)$ is the number of edges of G closer to u than v, and similarly $m_v(e|G)$ is the number of edges closer to v than to u.

A nanostructure is an intermediate nanoscale object obtained from engineering. Carbon nanomaterials are specific carbon allotropes with cylindrical or lattice molecular structures composed of carbon molecules. Topological indices are widely used in nanoscience to identify the structures and properties of nanocompounds. A C_4C_8 nanosheet is a trivalent decoration made by alternating squares C_4 and octagons C_8. There are two types of nanosheets which can be made by C_4 and C_8 following the trivalent decoration which we refer to as $T^1UC_4C_8[p,q]$ and $T^2UC_4C_8[p,q]$. The cut method is an important tool which uses different types of cuts to compute the distance based topological indices of benzenoid systems [13]. In this paper, we make use of the cut method in order to determine the explicit expressions of different bond additive indices of $T^1UC_4C_8[p,q]$-lattice, and $T^2UC_4C_8[p,q]$-lattice.

1 Main Results

Benzenoid systems are molecular graphs with benzenoid hydrocarbons. It is a finite, connected graph without cut vertices. A straight line segment C in the plane with end points

A and B is called a cut segment (or generally called cut) if C is orthogonal to one of the three edge directions, A and B the centers of edges, and the graph obtained by deleting all edges intersected by C has exactly two connected components. An elementary cut of a cut segment C is the set of all edges intersected by C [12]. Figure 1 indicates cut segments V_1, V_2, H_1, H_2, D_1 and D_2. In this section we obtain explicit expressions of bond additive indices of the of Mostar type of some classes of nanostructures.

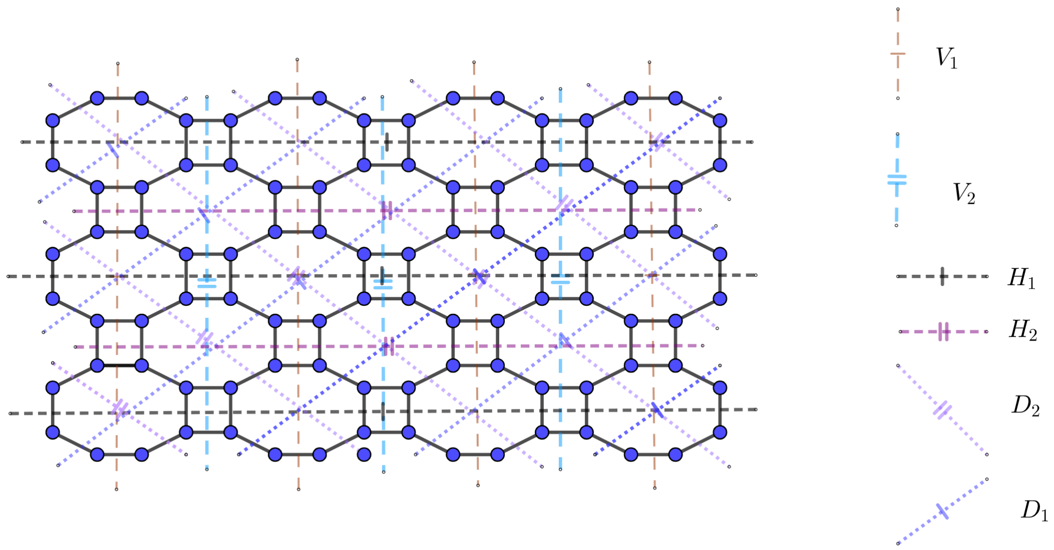

Figure 1. Different cuts of $T^1UC_4C_8[p,q]$.

Theorem 1.1. *For the graph $T^1UC_4C_8[p,q]$, the weighted edge versions of Mostar indices are as follows*

(a.) If p,q are even, then

$$\begin{aligned} MoA_e(T^1UC_4C_8[p,q]) &= 288p^2q^2 - 192p^2q + 4p^2 - 192pq^2 + 48pq + 4q^2 \\ &+ 2(45p^2q^2 - \frac{9p^4}{2} + 18p^3q + 18p^3 - 42p^2q - 22p^2 + 18pq^3 \\ &- 42pq^2 + 4pq + 8p - \frac{9q^4}{2} + 18q^3 - 22q^2 + 8q) - 4M, \end{aligned}$$

and

$$\begin{aligned} MoM_e(T^1UC_4C_8[p,q]) &= 432p^2q^2 - 312p^2q + 10p^2 - 312pq^2 + 72pq + 10q^2 \\ &+ 2(\frac{135p^2q^2}{2} - \frac{27p^4}{4} + 27p^3q + 30p^3 - 90p^2q - 39p^2 + 27pq^3 \\ &- 90pq^2 + 66pq + 12p - \frac{27q^4}{4} + 30q^3 - 39q^2 + 12q) - 8M. \end{aligned}$$

$$\text{where } M = \begin{cases} 12pq - 2q - 12p^2 + 2p, & \text{if } p < q \\ 12pq - 2p - 12q^2 + 2q, & \text{if } q < p \\ 0, & \text{if } p = q. \end{cases}$$

(b.) If p and q are both odd, then

$$\begin{aligned} MoA_e(T^1UC_4C_8[p,q]) &= 288p^2q^2 - 192p^2q + 4p^2 - 192pq^2 + 48pq + 24p + 4q^2 + 24q \\ &\quad - 8 + 2(45p^2q^2 - \frac{9p^4}{2} + 18p^3q + 18p^3 - 42p^2q - 22p^2 + 18pq^3 \\ &\quad - 42pq^2 + 4pq + 8p - \frac{9q^4}{2} + 18q^3 - 22q^2 + 8q) - 4M, \end{aligned}$$

and

$$\begin{aligned} MoM_e(T^1UC_4C_8[p,q]) &= 432p^2q^2 - 312p^2q + 10p^2 - 312pq^2 + 72pq + 60p + 10q^2 \\ &\quad + 60q - 20 + 2(\frac{135p^2q^2}{2} - \frac{27p^4}{4} + 27p^3q + 30p^3 - 90p^2q - 39p^2 \\ &\quad + 27pq^3 - 90pq^2 + 66pq + 12p - \frac{27q^4}{4} + 30q^3 - 39q^2 + 12q) \\ &\quad - 8M. \end{aligned}$$

(c.) If p is even and q is odd, then

$$\begin{aligned} MoA_e(T^1UC_4C_8[p,q]) &= 288p^2q^2 - 192p^2q + 4p^2 - 192pq^2 + 48pq + 24p + 4q^2 - 4 \\ &\quad + 2(45p^2q^2 - \frac{9p^4}{2} + 18p^3q - 24p^2q + 11p^2 + 18pq^3 - 24pq^2 \\ &\quad + 10pq - 4p - \frac{9q^4}{2} + 11q^2 - 4q - \frac{5}{2}) - 4M, \end{aligned}$$

and

$$\begin{aligned} MoM_e(T^1UC_4C_8[p,q]) &= 432p^2q^2 - 312p^2q + 10p^2 - 312pq^2 + 72pq + 60p + 10q^2 - 10 \\ &\quad + 2(\frac{135p^2q^2}{2} - \frac{27p^4}{4} + 27p^3q + 3p^3 - 63p^2q + \frac{39p^2}{2} + 27pq^3 \\ &\quad - 63pq^2 + 57pq - 15p - \frac{27q^4}{4} + 3q^3 + \frac{39q^2}{2} - 15q - \frac{3}{4}) - 8M. \end{aligned}$$

(d.) If p is odd and q is even, then

$$\begin{aligned} MoA_e(T^1UC_4C_8[p,q]) &= 288p^2q^2 - 192p^2q + 4p^2 - 192pq^2 + 48pq + 4q^2 + 24q - 4 \\ &\quad + 2(45p^2q^2 - \frac{9p^4}{2} + 18p^3q - 24p^2q + 11p^2 + 18pq^3 - 24pq^2 \\ &\quad + 10pq - 4p - \frac{9q^4}{2} + 11q^2 - 4q - \frac{5}{2}) - 4M, \end{aligned}$$

and

$$\begin{aligned} MoM_e(T^1UC_4C_8[p,q]) &= 432p^2q^2 - 312p^2q + 118p^2 - 312pq^2 + 72pq - 18p + 10q^2 \\ &+ 60q - 10 + 2(\frac{135p^2q^2}{2} - \frac{27p^4}{4} + 27p^3q + 3p^3 - 63p^2q + \frac{39p^2}{2} \\ &+ 27pq^3 - 63pq^2 + 57pq - 15p - \frac{27q^4}{4} + 3q^3 + \frac{39q^2}{2} - 15q - \frac{3}{4}) \\ &- 8M. \end{aligned}$$

Proof. We use a variant of cut method to determine the explicit expressions of the indices. We partition the edges of the graph into six different sets according to the different cuts on the graph. We use two different types of vertical cuts, namely V_1 and V_2 (see Figure 1) and two horizontal cuts, namely H_1 and H_2. Also, there are two different types of diagonal cuts, namely D_1 and D_2 (see Figure 1) to determine the expressions. Let M_1, M_2, M_3, M_4, M_5 and M_6 denote the contribution of the edges in the V_1, V_2, H_1, H_2, D_1 and D_2 cuts to the additively weighted Mostar index. Similarly, let $M'_1, M'_2, M'_3, M'_4, M'_5$ and M'_6 denote the contribution of the edges in the V_1, V_2, H_1, H_2, D_1 and D_2 cuts to the multiplicatively weighted Mostar index. Tables 1-2 gives the summary of the contribution of the edges in the vertical and horizontal cuts.

Table 1. Details regarding the contribution of the edges to the additively/multiplicatively weighted edge Mostar index for the edges in the vertical cuts.

Type of cut	No. of cuts	No. of edges in each cut	$\lvert m_u(e\lvert G) - m_v(e\lvert G)\rvert$	$d(u)+d(v)$	No. of edges
V_1	p	$2q$	$\lvert 12pq - 2p - 4q - 2(12qi - 8q - 2i + 1)\rvert$	4	2
V_1	p	$2q$	$\lvert 12pq - 2p - 4q - 2(12qi - 8q - 2i + 1)\rvert$	6	$2q-2$
V_2	$p-1$	$2q$	$\lvert 12pq - 2p - 4q - 2(12qi - 2q - 2i)\rvert$	6	$2q$
Type of cut	No. of cuts	No. of edges in each cut	$\lvert m_u(e\lvert G) - m_v(e\lvert G)\rvert$	$d(u)d(v)$	No. of edges
V_1	p	$2q$	$\lvert 12pq - 2p - 4q - 2(12qi - 8q - 2i + 1)\rvert$	4	2
V_1	p	$2q$	$\lvert 12pq - 2p - 4q - 2(12qi - 8q - 2i + 1)\rvert$	9	$2q-2$
V_2	$p-1$	$2q$	$\lvert 12pq - 2p - 4q - 2(12qi - 2q - 2i)\rvert$	9	$2q$

Table 2. Details regarding the contribution of the edges to the additively/multiplicatively weighted edge Mostar index for the edges in the horizontal cuts.

Type of cut	No. of cuts	No. of edges in each cut	$\lvert n_u(e\lvert G) - n_v(e\lvert G)\rvert$	$d(u)+d(v)$	No. of edges
H_1	q	$2p$	$\lvert 12pq - 2q - 4p - 2(12pi - 8p - 2i + 1)\rvert$	4	2
H_1	q	$2p$	$\lvert 12pq - 2q - 4p - 2(12pi - 8p - 2i + 1)\rvert$	6	$2p-2$
H_2	$q-1$	$2p$	$\lvert 12pq - 2q - 4p - 2(12pi - 2p - 2i)\rvert$	6	$2p$
Type of cut	No. of cuts	No. of edges in each cut	$\lvert n_u(e\lvert G) - n_v(e\lvert G)\rvert$	$d(u)d(v)$	No. of edges
H_1	q	$2p$	$\lvert 12pq - 2q - 4p - 2(12pi - 8p - 2i + 1)\rvert$	4	2
H_1	q	$2p$	$\lvert 12pq - 2q - 4p - 2(12pi - 8p - 2i + 1)\rvert$	9	$2p-2$
H_2	$q-1$	$2p$	$\lvert 12pq - 2q - 4p - 2(12pi - 2p - 2i)\rvert$	9	$2p$

Therefore, the contribution of the edges corresponding to the edges in V_1 cut is as follows. When p is even

$$\begin{aligned}
M_1 &= \sum_{e=uv\in V_1} (d(u)+d(v))|m_u(e|G)-m_v(e|G)| \\
&= 16\left(\sum_{i=1}^{\frac{p}{2}}(12pq-2p-4q-2(12qi-8q-2i+1))\right) \\
&+ 12\left(\sum_{i=1}^{\frac{p}{2}}(2q-2)(12pq-2p-4q-2(12qi-8q-2i+1))\right) \\
&= 8p^2(6q-1)+12p^2(6q^2-7q+1),
\end{aligned}$$

and

$$\begin{aligned}
M_1' &= \sum_{e=uv\in V_1} (d(u)d(v))|m_u(e|G)-m_v(e|G)| \\
&= 16\left(\sum_{i=2}^{\frac{p}{2}}(12pq-2p-4q-2(12qi-8q-2i+1))\right) \\
&+ 18\left(\sum_{i=1}^{\frac{p}{2}}(2q-2)(12pq-2p-4q-2(12qi-8q-2i+1))\right) \\
&= 8p^2(6q-1)+18p^2(6q^2-7q+1).
\end{aligned}$$

When p is odd

$$\begin{aligned}
M_1 &= \sum_{e=uv\in V_1} (d(u)+d(v))|m_u(e|G)-m_v(e|G)| \\
&= 16\left(\sum_{i=1}^{\frac{p-1}{2}}(12pq-2p-4q-2(12qi-8q-2i+1))\right) \\
&+ 12\left(\sum_{i=1}^{\frac{p-1}{2}}(2q-2)(12pq-2p-4q-2(12qi-8q-2i+1))\right) \\
&= 8(p^2-1)(6q-1)+12(p^2-1)(6q^2-7q+1),
\end{aligned}$$

and

$$\begin{aligned}
M_1' &= \sum_{e=uv\in V_1} (d(u)d(v))|m_u(e|G) - m_v(e|G)| \\
&= 16\left(\sum_{i=1}^{\frac{p-1}{2}} (12pq - 2p - 4q - 2(12qi - 8q - 2i + 1))\right) \\
&+ 18\left(\sum_{i=1}^{\frac{p-1}{2}} (2q-2)(12pq - 2p - 4q - 2(12qi - 8q - 2i + 1))\right) \\
&= 8(p^2-1)(6q-1) + 18(p^2-1)(6q^2-7q+1).
\end{aligned}$$

Similarly, the contribution of the V_2 cut, when p is even

$$\begin{aligned}
M_2 &= \sum_{e=uv\in V_1} (d(u)+d(v))|m_u(e|G) - m_v(e|G)| \\
&= 12\left(\sum_{i=1}^{\frac{p-2}{2}} 2q(12pq - 2p - 4q - 2(12qi - 2q - 2i)\right) \\
&= 12(p-2)pq(6q-1),
\end{aligned}$$

and

$$\begin{aligned}
M_2' &= \sum_{e=uv\in V_1} (d(u)d(v))|m_u(e|G) - m_v(e|G)| \\
&= 18\left(\sum_{i=1}^{\frac{p-2}{2}} 2q(12pq - 2p - 4q - 2(12qi - 2q - 2i))\right) \\
&= 18(p-2)pq(6q-1).
\end{aligned}$$

When p is odd

$$\begin{aligned}
M_2 &= \sum_{e=uv\in V1} (d(u)+d(v))|m_u(e|G) - m_v(e|G)| \\
&= 12\left(\sum_{i=1}^{\frac{p-1}{2}} 2q(12pq - 2p - 4q - 2(12qi - 2q - 2i))\right) \\
&= 12(p-1)^2q(6q-1),
\end{aligned}$$

and

$$\begin{aligned}
M_2' &= \sum_{e=uv\in V1} (d(u)d(v))|m_u(e|G) - m_v(e|G)| \\
&= 18\left(\sum_{i=1}^{\frac{p-1}{2}} 2q(12pq - 2p - 4q - 2(12qi - 2q - 2i))\right) \\
&= 18(p-1)^2q(6q-1).
\end{aligned}$$

Due to the symmetry, the contribution of the H_1, H_2 will be similar to the V_1, V_2 cuts. We can obtain this by replacing p by q and q by p. Therefore, when q is even

$$\begin{aligned} M_3 &= 8q^2(6p-1) + 12q^2(6p^2-7p+1), \\ M_3' &= 8q^2(6p-1) + 18q^2(6p^2-7p+1), \\ M_4 &= 12(q-2)pq(6p-1), \\ M_4' &= 18(q-2)pq(6p-1), \end{aligned}$$

and, when q is odd

$$\begin{aligned} M_3 &= 8(q^2-1)(6p-1) + 12(q^2-1)(6p^2-7p+1), \\ M_3' &= 8(q^2-1)(6p-1) + 18(q^2-1)(6p^2-7p+1), \\ M_4 &= 12(q-1)^2p(6p-1), \\ M_4' &= 18(q-1)^2p(6p-1). \end{aligned}$$

Now, we compute the contribution of the diagonal cuts. Due to the symmetry, the contribution of the diagonal cut D_1 will be same as D_2. Table 3 gives the contribution of the diagonal cuts D_1 and D_2.

Table 3. Details regarding the contribution of the edges to the additively/multiplicatively weighted edge Mostar index for the edges in the diagonal cuts.

Type of cut	No. of cuts	No. of edges in each cut	$\lvert m_u(e\lvert G) - m_v(e\lvert G)\rvert$	$d(u)+d(v)$	No. of edges
D_1	$p+q-1$	$2i$	$\lvert 12pq-2q-2p-2i-2(6i^2-3i)\rvert$	4	1 or 2
D_1	$p+q-1$	$2i$	$\lvert 12pq-2q-2p-2i-2(6i^2-3i)\rvert$	5	1 or 2
D_1	$p+q-1$	$2i$	$\lvert 12pq-2q-2p-2i-2(6i^2-3i)\rvert$	6	$2i-2$
Type of cut	No. of cuts	No. of edges in each cut	$\lvert m_u(e\lvert G) - m_v(e\lvert G)\rvert$	$d(u)d(v)$	No. of edges
D_1	$p+q-1$	$2i$	$\lvert 12pq-2q-2p-2i-2(6i^2-3i)\rvert$	4	1 or 2
D_1	$p+q-1$	$2i$	$\lvert 12pq-2q-2p-2i-2(6i^2-3i)\rvert$	6	1 or 2
D_1	$p+q-1$	$2i$	$\lvert 12pq-2q-2p-2i-2(6i^2-3i)\rvert$	9	$2i-2$

Let $p < q$.Then every $i-$th diagonal cut except the $p-$th and $(q-1)-$th cut contains exactly $2i-2$ edges having degree $d(u)+d(v)=6, (d(u)d(v)=9)$ and 2 edge having degree $d(u)+d(v)=5, (d(u)d(v)=6)$. In the case of $p-$th and $(q-1)-$th cuts there are $2i-2$ edges having degree $d(u)+d(v)=6, (d(u)d(v)=9)$ and one edge having degree $d(u)+d(v)=5, (d(u)d(v)=6)$ and the other edge have degree $d(u)+d(v)=4, (d(u)d(v)=4)$. When $p=q$, the middle diagonal cut contains exactly two edges with degree $d(u)+d(v)=4, (d(u)d(v)=4)$. Therefore, the contribution of the edges corresponding to the edges in D_1 cut is as follows.

When p and q are of the same parity with $p < q$,

$$\begin{aligned}
M_5 &= \sum_{e=uv\in D_1} (d(u)+d(v))|m_u(e|G)-m_v(e|G)| \\
&= 12\left(\sum_{i=1}^{\frac{p+q-2}{2}} (2i-2)(12pq-2p-2q-12i^2+4i)\right) \\
&+ 20\left(\sum_{i=1}^{\frac{p+q-2}{2}} (12pq-2p-2q-12i^2+4i)\right) - 2(12pq-2q-12p^2+2p) \\
&= 45p^2q^2 - \frac{9p^4}{2} + 18p^3q + 18p^3 - 42p^2q - 22p^2 + 18pq^3 - 42pq^2 + 4pq + 8p \\
&- \frac{9q^4}{2} + 18q^3 - 22q^2 + 8q - 2(12pq-2q-12p^2+2p).
\end{aligned}$$

Similarly, when p and q are of the same parity with $q < p$,

$$\begin{aligned}
M_5 &= 45p^2q^2 - \frac{9p^4}{2} + 18p^3q + 18p^3 - 42p^2q - 22p^2 + 18pq^3 - 42pq^2 + 4pq + 8p \\
&- \frac{9q^4}{2} + 18q^3 - 22q^2 + 8q - 2(12pq-2p-12q^2+2q).
\end{aligned}$$

When p and q are of the same parity with $p = q$, then

$$\begin{aligned}
M_5 &= 45p^2q^2 - \frac{9p^4}{2} + 18p^3q + 18p^3 - 42p^2q - 22p^2 + 18pq^3 - 42pq^2 + 4pq + 8p \\
&- \frac{9q^4}{2} + 18q^3 - 22q^2 + 8q.
\end{aligned}$$

Similarly, when p and q are of the same parity with $p < q$, one can deduce that

$$\begin{aligned}
M_5' &= \sum_{e=uv\in D_1} (d(u)d(v))|m_u(e|G)-m_v(e|G)| \\
&= 18\left(\sum_{i=1}^{\frac{p+q-2}{2}} (2i-2)(12pq-2p-2q-12i^2+4i)\right) \\
&+ 24\left(\sum_{i=1}^{\frac{p+q-2}{2}} (12pq-2p-2q-12i^2+4i)\right) - 4(12pq-2q-12p^2+2p) \\
&= \frac{135p^2q^2}{2} - \frac{27p^4}{4} + 27p^3q + 30p^3 - 90p^2q - 39p^2 + 27pq^3 - 90pq^2 + 66pq + 12p \\
&- \frac{27q^4}{4} + 30q^3 - 39q^2 + 12q - 4(12pq-2q-12p^2+2p).
\end{aligned}$$

When p and q are of the same parity with $q < p$,

$$M'_5 = \frac{135p^2q^2}{2} - \frac{27p^4}{4} + 27p^3q + 30p^3 - 90p^2q - 39p^2 + 27pq^3 - 90pq^2 + 66pq + 12p - \frac{27q^4}{4} + 30q^3 - 39q^2 + 12q - 4(12pq - 2p - 12q^2 + 2q).$$

When p, q are of same parity with $p = q$, we have

$$M'_5 = \frac{135p^2q^2}{2} - \frac{27p^4}{4} + 27p^3q + 30p^3 - 90p^2q - 39p^2 + 27pq^3 - 90pq^2 + 66pq + 12p - \frac{27q^4}{4} + 30q^3 - 39q^2 + 12q.$$

If p and q are of different parity and $p < q$, we have

$$\begin{aligned} M_5 &= \sum_{e=uv\in D_1} (d(u)+d(v))|m_u(e|G) - m_v(e|G)| \\ &= 12\left(\sum_{i=1}^{\frac{p+q-1}{2}} (2i-2)(12pq - 2p - 2q - 12i^2 + 4i)\right) \\ &+ 20\left(\sum_{i=1}^{\frac{p+q-1}{2}} (12pq - 2p - 2q - 12i^2 + 4i)\right) - 2(12pq - 2q - 12p^2 + 2p) \\ &= 45p^2q^2 - \frac{9p^4}{2} + 18p^3q - 24p^2q + 11p^2 + 18pq^3 - 24pq^2 + 10pq - 4p \\ &- \frac{9q^4}{2} + 11q^2 - 4q - \frac{5}{2} - 2(12pq - 2q - 12p^2 + 2p). \end{aligned}$$

When p and q are of different parity and $q < p$, we have

$$M_5 = -\frac{9p^4}{2} + 18p^3q - 24p^2q + 11p^2 + 18pq^3 - 24pq^2 + 10pq - 4p - \frac{9q^4}{2} + 11q^2 - 4q - \frac{5}{2} - 2(12pq - 2p - 12q^2 + 2q).$$

Similarly, if p and q are of different parity and $p < q$, we have

$$\begin{aligned}
M_5' &= \sum_{e=uv\in D_1} (d(u)d(v))|m_u(e|G) - m_v(e|G)| \\
&= 18\left(\sum_{i=1}^{\frac{p+q-1}{2}} (2i-2)(12pq - 2p - 2q - 12i^2 + 4i)\right) \\
&+ 24\left(\sum_{i=1}^{\frac{p+q-1}{2}} (12pq - 2p - 2q - 12i^2 + 4i)\right) - 2(12pq - 2q - 12p^2 + 2p) \\
&= \frac{135p^2q^2}{2} - \frac{27p^4}{4} + 27p^3q + 3p^3 - 63p^2q + \frac{39p^2}{2} + 27pq^3 - 63pq^2 + 57pq - 15p \\
&- \frac{27q^4}{4} + 3q^3 + \frac{39q^2}{2} - 15q - \frac{3}{4} - 4(12pq - 2q - 12p^2 + 2p),
\end{aligned}$$

and when p and q are of different parity and $q < p$, we have

$$\begin{aligned}
M_5' &= \frac{135p^2q^2}{2} - \frac{27p^4}{4} + 27p^3q + 3p^3 - 63p^2q + \frac{39p^2}{2} + 27pq^3 - 63pq^2 + 57pq - 15p \\
&- \frac{27q^4}{4} + 3q^3 + \frac{39q^2}{2} - 15q - \frac{3}{4} - 4(12pq - 2p - 12q^2 + 2q).
\end{aligned}$$

Now taking the different sums, we get the required result.

□

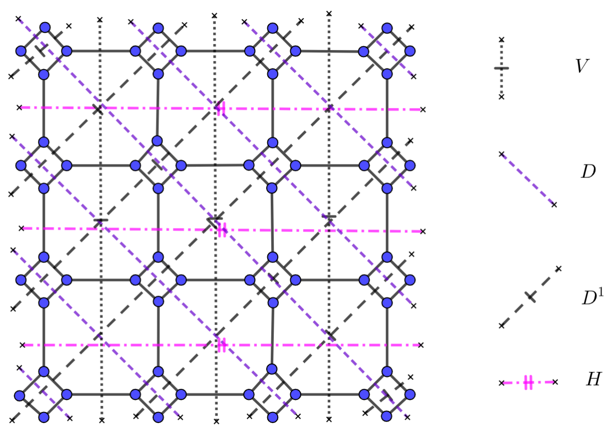

Figure 2. All the different cuts of the components $T^2UC_4C_8[p,q]$.

Theorem 1.2. *For the graph $T^2UC_4C_8[p,q]$, the weighted versions of Mostar indices are as follows* *(a.) If p and q are even, then*

$$MoA_e(T^2UC_4C_8[p,q]) = 36p^2q^2 + 33p^2q + 15p^2 + 33pq^2 + 15q^2$$
$$+ 10(-p^3 + p^2(9q+8) + p(9q^2+16q+8) + q(-q^2+8q+8))$$
$$+ \frac{1}{2}(-9p^4 + 4p^3(9q+14) + 6p^2(15q^2+4q-6) + 4p(9q^3+6q^2-18q-20)$$
$$- q(9q^3 - 56q^2 + 36q + 80)) - 4M',$$

and

$$MoM_e(T^2UC_4C_8[p,q]) = 54p^2q^2 + \frac{99p^2q}{2} + \frac{45p^2}{2} + \frac{99pq^2}{2} + \frac{45q^2}{2}$$
$$+ 12(-p^3 + p^2(9q+8) + p(9q^2+16q+8) + q(-q^2+8q+8))$$
$$+ \frac{3}{4}(-9p^4 + 4p^3(9q+14) + 6p^2(15q^2+4q-6) + 4p(9q^3+6q^2-18q-20)$$
$$- q(9q^3 - 56q^2 + 36q + 80)) - 8M'.$$

$$\text{where } M' = \begin{cases} (6pq + 7p + 5q + 4 - 6p^2), & \text{if } p < q \\ (6pq + 7q + 5p + 4 - 6q^2), & \text{if } q < p \\ 0, & \text{if } p = q. \end{cases}$$

(b.) If p and q are odd, then

$$Mo_A(T^2UC_4C_8[p,q]) = 36p^2q^2 + 33p^2q - 3p^2 + 33pq^2 - 33p - 3q^2 - 33q - 30$$
$$+ 10(-p^3 + p^2(9q+8) + p(9q^2+16q+8) + q(-q^2+8q+8))$$
$$+ \frac{1}{2}(-9p^4 + 4p^3(9q+14) + 6p^2(15q^2+4q-6) + 4p(9q^3+6q^2-18q-20)$$
$$- q(9q^3 - 56q^2 + 36q + 80)) - 4M',$$

and

$$MoM_e(T^2UC_4C_8[p,q]) = 54p^2q^2 + \frac{99p^2q}{2} - \frac{9p^2}{2} + \frac{99pq^2}{2} - \frac{99p}{2} - \frac{9q^2}{2} - \frac{99q}{2} - 45$$
$$+ 12(-p^3 + p^2(9q+8) + p(9q^2+16q+8) + q(-q^2+8q+8))$$
$$+ \frac{3}{4}(-9p^4 + 4p^3(9q+14) + 6p^2(15q^2+4q-6) + 4p(9q^3+6q^2-18q-20)$$
$$- q(9q^3 - 56q^2 + 36q + 80)) - 8M'.$$

(c.) If p is even and q is odd, then

$$Mo_A(T^2UC_4C_8[p,q]) = 36p^2q^2 + 33p^2q - 3p^2 + 33pq^2 - 33p + 15q^2 - 15$$
$$+ 10(-p^3 + p^2(9q+5) + p(9q^2+22q+11) - q^3 + 5q^2 + 11q + 5)$$
$$+ \frac{1}{2}(-9p^4 + 4p^3(9q+5) + 6p^2(15q^2+10q+3) + 4p(9q^3+15q^2-9q-5)$$
$$- 9q^4 + 20q^3 + 18q^2 - 20q - 9) - 4M',$$

and

$$MoM_e(T^2UC_4C_8[p,q]) = 54p^2q^2 + \frac{99p^2q}{2} - \frac{9p^2}{2} + \frac{99pq^2}{2} - \frac{99p}{2} + \frac{45q^2}{2} - \frac{45}{2}$$
$$+ 12(-p^3 + p^2(9q+5) + p(9q^2 + 22q + 11) - q^3 + 5q^2 + 11q + 5)$$
$$+ \frac{3}{4}(-9p^4 + 36p^3q + 20p^3 + 90p^2q^2 + 60p^2q + 18p^2 + 36pq^3 + 60pq^2 - 36pq - 20p$$
$$- 9q^4 + 20q^3 + 18q^2 - 20q - 9) - 8M'.$$

(d.) If p is odd and q is even, then

$$Mo_A(T^2UC_4C_8[p,q]) = 36p^2q^2 + 33p^2q + 15p^2 + 33pq^2 - 3q^2 - 33q - 15$$
$$+ 10(-p^3 + p^2(9q+5) + p(9q^2 + 22q + 11) - q^3 + 5q^2 + 11q + 5)$$
$$+ \frac{1}{2}(-9p^4 + 4p^3(9q+5) + 6p^2(15q^2 + 10q + 3) + 4p(9q^3 + 15q^2 - 9q - 5) - 9q^4$$
$$+ 20q^3 + 18q^2 - 20q - 9) - 4M',$$

and

$$MoM_e(T^2UC_4C_8[p,q]) = 54p^2q^2 + \frac{99p^2q}{2} + \frac{45p^2}{2} + \frac{99pq^2}{2} - \frac{9q^2}{2} - \frac{99q}{2} - \frac{45}{2}$$
$$+ 12(-p^3 + p^2(9q+5) + p(9q^2 + 22q + 11) - q^3 + 5q^2 + 11q + 5) + \frac{3}{4}(-9p^4 + 36p^3q$$
$$+ 20p^3 + 90p^2q^2 + 60p^2q + 18p^2 + 36pq^3 + 60pq^2 - 36pq$$
$$- 20p - 9q^4 + 20q^3 + 18q^2 - 20q - 9) - 8M'.$$

Proof. We use a variant of the cut method to determine the explicit expressions of the indices. We partition the edges of the graph into four different sets according to the different cuts on the graph. We consider one horizontal cut H, one vertical cut V and two diagonal cuts D and D^1, respectively. Let C_1, C_2, C_3 and C_4 denote the sum corresponding to the cuts H, V, D and D^1, respectively for the additively weighted edge Mostar index. Similarly, let C_1', C_2', C_3' and C_4' denote the sum corresponding to the cuts H, V, D and D^1, respectively for the multiplicatively weighted edge Mostar index. Table 4 gives the summary of the contribution of the edges in different cuts.

Therefore, the contribution of the edges corresponding to the edges in H and V cut is as follows. When p is even, we have

$$C_1 = \sum_{e=uv \in V} (d(u) + d(v))|m_u(e|G) - m_v(e|G)|$$
$$= 12(q+1)\left(\sum_{i=1}^{\frac{p}{2}} (6pq + 5p + 5q + 4 - (q+1) - 2((5q+4)i + (i-1)(q+1)))\right)$$
$$= 18p^2q^2 + 33p^2q + 15p^2,$$

Table 4. Details regarding the contribution of the edges to the additively/multiplicatively weighted Mostar index for the edges for different cuts.

		Vertical cut V		
No. of cuts	No. of edges in each cut	$\|m_u(e\|G) - m_v(e\|G)\|$	$d(u)+d(v)$	No. of edges
p	$q+1$	$\|6pq+5p+5q+4-(q+1)-2((5q+4)i+(i-1)(q+1))\|$	6	$q+1$
No. of cuts	No. of edges in each cut	$\|m_u(e\|G) - m_v(e\|G)\|$	$d(u)d(v)$	No. of edges
q	$p+1$	$\|6pq+5p+5q+4-(q+1)-2((5q+4)i+(i-1)(q+1))\|$	9	$q+1$
		Horizontal cut H		
No. of cuts	No. of edges in each cut	$\|m_u(e\|G) - m_v(e\|G)\|$	$d(u)+d(v)$	No. of edges
q	$p+1$	$\|6pq+5p+5q+4-(q+1)-2((5p+4)i+(i-1)(p+1))\|$	6	$p+1$
No. of cuts	No. of edges in each cut	$\|m_u(e\|G) - m_v(e\|G)\|$	$d(u)d(v)$	No. of edges
q	$p+1$	$\|6pq+5p+5q+4-(q+1)-2((5p+4)i+(i-1)(p+1))\|$	9	$p+1$
		Diagonal cut D/D^1		
No. of cuts	No. of edges in each cut	$\|m_u(e\|G) - m_v(e\|G)\|$	$d(u)+d(v)$	No. of edges
$p+q+1$	$2i$	$\|6pq+5p+5q+4-6i^2+2i\|$	5	1 or 2
$p+q+1$	$2i$	$\|6pq+5p+5q+4-6i^2+2i\|$	4	1 or 2
$p+q+1$	$2i$	$\|6pq+5p+5q+4-6i^2+2i\|$	6	$2i-2$
No. of cuts	No. of edges in each cut	$\|m_u(e\|G) - m_v(e\|G)\|$	$d(u)d(v)$	No. of edges
$p+q+1$	$2i$	$\|6pq+5p+5q+4-6i^2+2i\|$	6	1 or 2
$p+q+1$	$2i$	$\|6pq+5p+5q+4-6i^2+2i\|$	4	1 or 2
$p+q+1$	$2i$	$\|6pq+5p+5q+4-6i^2+2i\|$	9	$2i-2$

and

$$
\begin{aligned}
C_1' &= \sum_{e=uv\in V} (d(u)+d(v))|m_u(e|G) - m_v(e|G)| \\
&= 18(q+1)\left(\sum_{i=1}^{\frac{p}{2}} (6pq+5p+5q+4-(q+1)-2((5q+4)i+(i-1)(q+1)))\right) \\
&= 27p^2q^2 + \frac{99p^2q}{2} + \frac{45p^2}{2}.
\end{aligned}
$$

When p is odd, one can see that

$$
\begin{aligned}
C_1 &= \sum_{e=uv\in V} (d(u)+d(v))|n_u(e|G) - n_v(e|G)| \\
&= 12(q+1)\left(\sum_{i=1}^{\frac{p-1}{2}} (6pq+5p+5q+4-(q+1)-2((5q+4)i+(i-1)(q+1)))\right) \\
&= 18p^2q^2 + 33p^2q + 15p^2 - 18q^2 - 33q - 15,
\end{aligned}
$$

and

$$\begin{aligned}
C_1' &= \sum_{e=uv\in V} (d(u)+d(v))|n_u(e|G)-n_v(e|G)| \\
&= 18(q+1)\left(\sum_{i=1}^{\frac{p-1}{2}}(6pq+5p+5q+4-(q+1)-2((5q+4)i+(i-1)(q+1)))\right) \\
&= 27p^2q^2+\frac{99p^2q}{2}+\frac{45p^2}{2}-27q^2-\frac{99q}{2}-\frac{45}{2}.
\end{aligned}$$

From the previous expressions, we can compute the contribution of horizontal cut by replacing p and q by q and p, respectively. Therefore, when q is even

$$\begin{aligned}
C_2 &= 18p^2q^2+33q^2p+15q^2, \\
C_2' &= 27p^2q^2+\frac{99q^2p}{2}+\frac{45q^2}{2}.
\end{aligned}$$

When q is odd, we obtain

$$\begin{aligned}
C_2 &= 18p^2q^2+33q^2p+15q^2-18p^2-33p-15, \\
C_2' &= 27p^2q^2+\frac{99q^2p}{2}+\frac{45q^2}{2}-27p^2-\frac{99p}{2}-\frac{45}{2}.
\end{aligned}$$

Due to symmetry of the structure, the total number of diagonal cuts of D and D^1 must be the same. Hence the total contribution of both the diagonal cuts must be the same. Thus, the contribution when p, q are of the the same parity with $p<q$ is

$$\begin{aligned}
C_3 = C_4 &= \sum_{e=uv\in D} (d(u)+d(v))|n_u(e|G)-n_v(e|G)| \\
&= 20\left(\sum_{i=1}^{\frac{p+q}{2}}(6pq+5p+5q+4-6i^2+2i)\right) \\
&+ 12\left(\sum_{i=1}^{\frac{p+q}{2}}(2i-2)(6pq+5p+5q+4-6i^2+2i)\right)-2(6pq+7p+5q+4-6p^2) \\
&= 5(-p^3+p^2(9q+8)+p(9q^2+16q+8)+q(-q^2+8q+8)) \\
&+ \frac{1}{4}(-9p^4+4p^3(9q+14)+6p^2(15q^2+4q-6)+4p(9q^3+6q^2-18q-20) \\
&- q(9q^3-56q^2+36q+80))-2(6pq+7p+5q+4-6p^2).
\end{aligned}$$

When $q<p$, we have

$$\begin{aligned}
C_3 = C_4 &= 5(-p^3+p^2(9q+8)+p(9q^2+16q+8)+q(-q^2+8q+8)) \\
&+ \frac{1}{4}(-9p^4+4p^3(9q+14)+6p^2(15q^2+4q-6)+4p(9q^3+6q^2-18q-20) \\
&- q(9q^3-56q^2+36q+80))-2(6pq+7q+5p+4-6q^2),
\end{aligned}$$

and, when $p = q$

$$\begin{aligned} C_3 = C_4 &= 5(-p^3 + p^2(9q+8) + p(9q^2+16q+8) + q(-q^2+8q+8)) \\ &+ \frac{1}{4}(-9p^4 + 4p^3(9q+14) + 6p^2(15q^2+4q-6) + 4p(9q^3+6q^2-18q-20) \\ &- q(9q^3-56q^2+36q+80)). \end{aligned}$$

When p and q are of different parity with $p < q$

$$\begin{aligned} C_3 = C_4 &= \sum_{e=uv\in D_1} (d(u)+d(v))|m_u(e|G) - m_v(e|G)| \\ &= 20\left(\sum_{i=1}^{\frac{p+q+1}{2}} (6pq+5p+5q+4-6i^2+2i)\right) \\ &+ 12\left(\sum_{i=1}^{\frac{p+q+1}{2}} ((2i-2)(6pq+5p+5q+4-6i^2+2i))\right) - 2(6pq+7p+5q+4-6p^2) \\ &= 5(-p^3 + p^2(9q+5) + p(9q^2+22q+11) - q^3 + 5q^2 + 11q + 5) \\ &- 2(6pq+7p+5q+4-6p^2) + \frac{1}{4}(-9p^4 + 4p^3(9q+5) + 6p^2(15q^2+10q+3) \\ &+ 4p(9q^3+15q^2-9q-5) - 9q^4 + 20q^3 + 18q^2 - 20q - 9). \end{aligned}$$

When p and q are of different parity with $q < p$

$$\begin{aligned} C_3 = C_4 &= 5(-p^3 + p^2(9q+5) + p(9q^2+22q+11) - q^3 + 5q^2 + 11q + 5) \\ &- 2(6pq+7q+5p+4-6q^2) + \frac{1}{4}(-9p^4 + 4p^3(9q+5) + 6p^2(15q^2+10q+3) \\ &+ 4p(9q^3+15q^2-9q-5) - 9q^4 + 20q^3 + 18q^2 - 20q - 9). \end{aligned}$$

Similarly, when p and q are of the same parity with $p < q$

$$\begin{aligned} C_3' = C_4' &= \sum_{e=uv\in D} (d(u)+d(v))|n_u(e|G) - n_v(e|G)| \\ &= 24\left(\sum_{i=1}^{\frac{p+q}{2}} (6pq+5p+5q+4-6i^2+2i)\right) \\ &+ 18\left(\sum_{i=1}^{\frac{p+q}{2}} (2l-2)(6pq+5p+5q+4-6i^2+2i)\right) - 4(6pq+7p+5q+4-6p^2) \\ &= 6(-p^3 + p^2(9q+8) + p(9q^2+16q+8) + q(-q^2+8q+8)) + \frac{3}{8}(-9p^4 \\ &+ 4p^3(9q+14) + 6p^2(15q^2+4q-6) + 4p(9q^3+6q^2-18q-20) \\ &- q(9q^3-56q^2+36q+80)) - 4(6pq+7p+5q+4-6p^2). \end{aligned}$$

When p and q are of the same parity with $q < p$

$$\begin{aligned}C_3' = C_4' &= 6(-p^3 + p^2(9q+8) + p(9q^2+16q+8) + q(-q^2+8q+8)) \\ &+ \frac{3}{8}(-9p^4 + 4p^3(9q+14) + 6p^2(15q^2+4q-6) + 4p(9q^3+6q^2-18q-20) \\ &- q(9q^3-56q^2+36q+80)) - 4(6pq+7q+5p+4-6q^2).\end{aligned}$$

When p and q are of the same parity with $p = q$

$$\begin{aligned}C_3' = C_4' &= 6(-p^3 + p^2(9q+8) + p(9q^2+16q+8) + q(-q^2+8q+8)) \\ &+ \frac{3}{8}(-9p^4 + 4p^3(9q+14) + 6p^2(15q^2+4q-6) + 4p(9q^3+6q^2-18q-20) \\ &- q(9q^3-56q^2+36q+80)).\end{aligned}$$

If p and q are of different parity with $p < q$, then

$$\begin{aligned}&C_3' = C_4' = \sum_{e=uv\in D} (d(u)+d(v))|m_u(e|G) - m_v(e|G)| \\ &= 24\left(\sum_{i=1}^{\frac{p+q+1}{2}} (6pq+5p+5q+4-6i^2+2i)\right) \\ &+ 18\left(\sum_{i=1}^{\frac{p+q+1}{2}} ((2i-2)(6pq+5p+5q+4-6i^2+2i))\right) - 4(6pq+7p+5q+4-6p^2) \\ &= 6(-p^3 + p^2(9q+5) + p(9q^2+22q+11) - q^3 + 5q^2 + 11q + 5) + \frac{3}{8}(-9p^4 + 36p^3q + 20p^3 \\ &+ 90p^2q^2 + 60p^2q + 18p^2 + 36pq^3 + 60pq^2 - 36pq - 20p - 9q^4 + 20q^3 + 18q^2 - 20q - 9) \\ &- 4(6pq+7p+5q+4-6p^2).\end{aligned}$$

Finally, if p and q are of different parity with $q < p$, then

$$\begin{aligned}&C_3' = C_4' = 6(-p^3 + p^2(9q+5) + p(9q^2+22q+11) - q^3 + 5q^2 + 11q + 5) \\ &+ \frac{3}{8}(-9p^4 + 36p^3q + 20p^3 + 90p^2q^2 + 60p^2q + 18p^2 + 36pq^3 + 60pq^2 - 36pq - 20p - 9q^4 \\ &+ 20q^3 + 18q^2 - 20q - 9) - 4(6pq+7q+5p+4-6q^2).\end{aligned}$$

Now collecting the different sums, we get the required result. □

2 Conclusion

In this paper, we computed the explicit expressions of the weighted Mostar type bond additive indices of certain class of carbon nanostructures using a variant of the cut method. There are several other carbon nanostructures in which the bond additive indices have not been explored, which is a problem which needs further research.

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Data Availability Statement

Data is contained within the article.

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Conflicts of Interests

The authors declare that they have no conflicts of interest regarding the publication of this article.

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