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On the Cayleyness of bipartite Kneser graphs

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Abstract. For any given $n,k \in \mathbb{N}$ with $2k < n$, the bipartite Kneser graph $H(n,k)$ is defined as the graph whose vertex set is the family of k-subsets and $(n-k)$-subsets of $[n] = \{1,2,\ldots,n\}$ in which any two vertices are adjacent if and only if one of them is a subset of the other. In this paper, we study some algebraic properties of the bipartite Kneser graph $H(n,k)$. In particular, we determine the values of n,k for which the bipartite Kneser graph $H(n,k)$ is a Cayley graph.

Keywords: bipartite Kneser graph, vertex-transitive graph, automorphism group, Cayley graph.
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1 Introduction

For a positive integer $n > 1$, let $[n] = \{1,2,\ldots,n\}$ and V be the set of all k-subsets and $(n-k)$-subsets of $[n]$. The *bipartite Kneser graph* $H(n,k)$ has V as its vertex set, and vertices A,B are adjacent if and only if $A \subset B$ or $B \subset A$. If $n = 2k$, it is obvious that we do not have any edges, and in such a case, $H(n,k)$ is a null graph, and hence we assume that $n \geq 2k+1$. It follows from the definition of the graph $H(n,k)$, that it has $2\binom{n}{k}$ vertices and the degree of each of its vertex is $\binom{n-k}{k}$= $\binom{n-k}{n-2k}$, hence it is a regular graph. It is clear that $H(n,k)$ is a bipartite graph. In fact, if $V_1 = \{v \in V(H(n,k))| \ |v| = k\}$ and $V_2 = \{v \in V(H(n,k))| \ |v| = n-k\}$, then $\{V_1, V_2\}$ is a partition of $V(H(n,k))$ and every edge of $H(n,k)$ has a vertex in V_1 and a vertex in V_2 and $|V_1| = |V_2|$.

It is easy to show that the graph $H(n,k)$ is a connected graph. The bipartite Kneser graph

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$H(2n-1, n-1)$ is known as the middle cube MQ_{2n-1} [19,27] or regular hyper-star graph $HS(2n, n)$ [11,13,16,18]. The graph MQ_{2n-1} has been studied by various authors and some of the papers about it are [11,13,19,20]. Figure 1. shows the bipartite Kneser graph $H(5,2)$ in the plane. Note that in Figure1, the set $\{i, j, k\}$ ($\{i, j\}$) is denoted by ijk (ij).

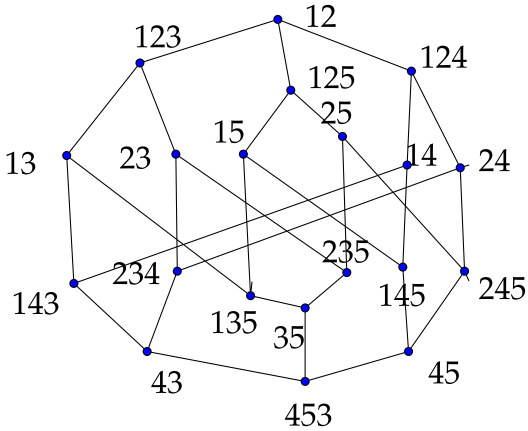

Figure 1. Bipartite Kneser graph $H(5,2)$.

In this paper, among various interesting properties of the bipartite Kneser graph $H(n, k)$, we interested in its automorphism group. We want to show how this group acts on the vertex set. Mirafzal [13] determined the automorphism group of $MQ_{2n-1} = HS(2n, n) = H(2n-1, n-1)$ and showed that $HS(2n, n)$ is a vertex-transitive non Cayley graph. Also, he showed that $H(2n-1, n-1)$ is arc-transitive. Later, he determined the automorphism group of the graph $H(n, k)$ and show that this graph is a vertex-transitive graph for all values of n, k [16,20]. We think that the following question is an unanswered question;

Question For what values of n, k the bipartite Kneser graph $H(n, k)$ is a Cayley graph?

In this paper, we answer the question.

2 Preliminaries

In this paper, a graph $\Gamma = (V, E)$ is considered as a finite undirected simple graph, where $V = V(\Gamma)$ is the vertex set and $E = E(\Gamma)$ is the edge set. For all the terminology and notation not defined here, we follow [4,5,7].

The graphs $\Gamma_1 = (V_1, E_1)$ and $\Gamma_2 = (V_2, E_2)$ are called *isomorphic*, if there is a bijection $\alpha : V_1 \longrightarrow V_2$ such that $\{a, b\} \in E_1$ if and only if $\{\alpha(a), \alpha(b)\} \in E_2$ for all $a, b \in V_1$. In such a case the bijection α is called an isomorphism. An *automorphism* of a graph Γ is an isomorphism of Γ with itself. The set of automorphisms of Γ with the operation of composition of functions is a group, called the *automorphism group* of Γ and denoted by $Aut(\Gamma)$.

The group of all permutations of a set V is denoted by $Sym(V)$ or just $Sym(n)$ when

$|V| = n$. A *permutation group* G on V is a subgroup of $Sym(V)$. In this case we say that G acts on V. If X is a graph with vertex set V, then we can view each automorphism as a permutation of V, and so $Aut(X)$ is a permutation group. If G acts on V, we say that G is *transitive* (or G acts *transitively* on V), when there is just one orbit. This means that given any two elements u and v of V, there is an element β of G such that $\beta(u) = v$.

The graph Γ is called *vertex-transitive* if $Aut(\Gamma)$ acts transitively on $V(\Gamma)$. For $v \in V(\Gamma)$ and $G = Aut(\Gamma)$, the stabilizer subgroup G_v is the subgroup of G consisting of all automorphisms that fix v. In the vertex-transitive case all stabilizer subgroups G_v are conjugate in G, and consequently isomorphic. In this case, the index of G_v in G is given by the equation, $|G : G_v| = \frac{|G|}{|G_v|} = |V(\Gamma)|$. If each stabilizer G_v is the identity group, then every element of G, except the identity, does not fix any vertex, and we say that G acts semiregularly on V. We say that G acts regularly on V if and only if G acts transitively and semiregularly on V and in this case we have $|V| = |G|$.

Let G be any abstract finite group with identity 1, and suppose Ω is a set of G, with the properties:

(i) $x \in \Omega \Longrightarrow x^{-1} \in \Omega$; (ii) $1 \notin \Omega$.

The *Cayley graph* $\Gamma = \Gamma(G;\Omega)$ is the (simple) graph whose vertex-set and edge-set defined as follows :

$V(\Gamma) = G$, $E(\Gamma) = \{\{g,h\} \mid g^{-1}h \in \Omega\}$. It can be shown that a connected graph Γ is a Cayley graph if and only if $Aut(\Gamma)$ contains a subgroup H, such that H acts regularly on $V(\Gamma)$ [4, chap 16].

3 Main results

Proposition 3.1. *The graph $H(n,k)$ is a vertex-transitive graph.*

Proof. Let $[n] = \{1,2,\ldots,n\}$, $\Gamma = H(n,k)$ and $V = V(\Gamma)$. It is easy to prove that the graph Γ is a regular bipartite graph. In fact, if $V_1 = \{v \in V| \, |v| = k\}$ and $V_2 = \{v \in V| \, |v| = n-k\}$, then $V = V_1 \cup V_2$ and $|V_1| = |V_2|=\binom{n}{k}$, and every edge of Γ has a vertex in V_1 and a vertex in V_2. Suppose $u,v \in V$. In the following steps, we show that Γ is a vertex-transitive graph.

(i) If both vertices u and v lie in V_1 and $|u \cap v| = t$, then we may assume

$$u = \{x_1,\ldots,x_t,u_1,\ldots,u_{k-t}\}$$

and $v = \{x_1,\ldots,x_t,v_1,\ldots,v_{k-t}\}$, where $x_i,u_j,v_h \in [n]$. Let σ be a permutation of $Sym([n])$ such that $\sigma(x_i) = x_i$, $\sigma(u_i) = v_i$ and $\sigma(w_j) = w_j$, where $w_j \in [n] - (u \cup v)$. Therefore, σ induces an automorphism $f_\sigma : V(\Gamma) \to V(\Gamma)$ by the rule,

$$f_\sigma\{x_1,\ldots,x_t,u_1,\ldots,u_{k-t}\} = \{\sigma(x_1),...,\sigma(x_t),\sigma(u_1),\ldots,\sigma(u_{k-t})\}.$$

Hence $f_\sigma(u) = v$.

(ii) We now assume that both vertices u and v lie in V_2. We can see by an easy argument that the mapping $\alpha : V(\Gamma) \to V(\Gamma)$, defined by the rule $\alpha(v) = v^c$, where v^c is the complement of the set v in $[n]$ (for every v in V), is an automorphism of Γ. Therefore $\alpha(u), \alpha(v) \in V_1$, and hence there is an automorphism f_σ in $Aut(\Gamma)$ such that $f_\sigma(\alpha(u)) = \alpha(v)$. Thus $(\alpha^{-1} f_\sigma \alpha)(u) = v$, where $(\alpha^{-1} f_\sigma \alpha) \in Aut(\Gamma)$.

(iii) Now let $u \in V_1$ and $v \in V_2$, thus $\alpha(v) \in V_1$ and hence there is an automorphism f_σ in $Aut(\Gamma)$ such that $f_\sigma(u) = \alpha(v)$, thus $(\alpha^{-1} f_\sigma)(u) = v$. □

The study of vertex-transitive graphs has a long and rich history in discrete mathematics. Prominent examples of vertex-transitive graphs are Cayley graphs which are important in both theory as well as applications. Some of the recent studies on Cayley graphs include [2,10,12]. Vertex-transitive graphs that are not Cayley graphs, for which we use the abbreviation VTNCG, have been an object of a systematic study since 1980 [3,6]. In trying to recognize whether or not a vertex-transitive graph is a Cayley graph, we are left with the problem of determining whether the automorphism group contains a regular subgroup [4. chapter 16]. The reference [1] is an excellent source for studying graphs that are VTNCG. In particular, determining the automorphism group of a given graph can be very useful in determining whether this graph is a Cayley graph. Although, to find the automorphism group of a graph may be difficult, but it is one of the research area in algebraic graph theory. Some of the recent works include [13-26]. On the automorphism group of the graph $H(n,k)$, we have the following result.

Theorem 3.2. [16] *Let n and k be integers with $\frac{n}{2} > k \geq 1$, and let $\Gamma = (V,E) = H(n,k)$ be a bipartite Kneser graph with partition $V = V_1 \cup V_2$, $V_1 \cap V_2 = \emptyset$, where $V_1 = \{v \mid v \subset [n], |v| = k\}$ and $V_2 = \{w \mid w \subset [n], |w| = n-k\}$. Then $Aut(\Gamma) \cong Sym([n]) \times \mathbb{Z}_2$, where $\mathbb{Z}_2$ is the cyclic group of order 2.*

We now want to investigate Cayley properties of the bipartite Kneser graph $H(n,k)$. In the first step, we show that if $k = 1$, then $H(n,k)$ is a Cayley graph.

Proposition 3.3. [18] *The bipartite Kneser graph $H(n,1)$ is a Cayley graph.*

Proof. Let $[n] = \{1,2,...,n\}$, $\Gamma = H(n,1)$ and $\Lambda = Cay(\mathbb{D}_{2n}, \Omega)$, where $\mathbb{D}_{2n} = \langle a,b \mid a^n = b^2 = 1, ba = a^{n-1}b \rangle$ is the dihedral group of order $2n$, and $\Omega = \{ab, a^2b, ..., a^{n-1}b\}$. Note that Ω is an inverse-closed subset of $\mathbb{D}_{2n} - \{1\}$ (note that $(a^i b)^2 = 1$). We show that Λ is isomorphic to the graph $H(n,1)$. Consider the following mapping f;

$$f : V(\Gamma) \longrightarrow V(\Lambda)$$

$$f(v) = \begin{cases} a^i & v = \{i\}, i \in [n] \\ a^j b & v = [n] - \{j\}, j \in [n]. \end{cases}$$

It is is clear that f is a bijective mapping. Let $\{i\}$ and $[n] - \{j\}$ be two vertices of Γ, then

$$\{i\} \leftrightarrow [n] - \{j\} \Leftrightarrow \{i\} \subset [n] - \{j\} \Leftrightarrow i \neq j$$

$$\Leftrightarrow (a^i)^{-1}a^j b \in \Omega \Leftrightarrow a^i \leftrightarrow a^j b.$$

Note that if $i = j$ then $(a^i)^{-1}a^j b = b \notin \Omega$. Therefore $H(n,1) \cong \Lambda = Cay(\mathbb{D}_{2n},\Omega)$. □

4 Direct products of graphs

If Γ_1, Γ_2 are graphs, then their direct product is the graph $\Gamma_1 \times \Gamma_2$ with vertex set

$$\{(v_1,v_2) \mid v_1 \in \Gamma_1, v_2 \in \Gamma_2\}$$

and for which vertices (v_1,v_2) and (w_1,w_2) are adjacent precisely if v_1 is adjacent to w_1 in Γ_1 and v_2 is adjacent to w_2 in Γ_2. It can be shown that the direct product is commutative and associative [8]. The following theorem, first proved by Weichsel (1962) characterizes connectedness in direct products of two factors [8].

Theorem 4.1. [8] *Suppose Γ_1 and Γ_2 are connected nontrivial graphs. If at least one of Γ_1 or Γ_2 has an odd cycle, then $\Gamma_1 \times \Gamma_2$ is connected. If both Γ_1 and Γ_2 are bipartite, then $\Gamma_1 \times \Gamma_2$ has exactly two components.*

We need the following theorem in the sequel.

Theorem 4.2. *Let G_1 and G_2 be groups, $S_1 \subset G_1$, $S_2 \subset G_2$, $S_1 = {S_1}^{-1}, S_2 = {S_2}^{-1}$ and $1 \notin S_1, 1 \notin S_2$. Then*

$$Cay(G_1,S_1) \times Cay(G_1,S_1) = Cay(G_1 \times G_2, S_1 \times S_2)$$

where $G_1 \times G_2$ is the direct product of groups G_1 and G_2.

Proof. The proof is straightforward.

□

We recall that, the *Kneser graph $K(n,k)$* is the graph with the familiy of k-subsets of $[n]$ as its vertex-set, in which two vertices v,w are agjacent when $v \cap w = \emptyset$. We now want to investigate Cayley properties of the bipartite Kneser graph $H(n,k)$. In the first step, we show that if $k = 1$, then $H(n,k)$ is a Cayley graph.

Theorem 4.3. *Let n,k be positive integers and K_2 be the complete graph on the set $\{0,1\}$. Then for the bipartite Kneser graph $H(n,k)$, we have*

$$H(n,k) \cong K(n,k) \times K_2.$$

Proof. We define the mapping $f : V(H(n,k)) \rightarrow V(K(n,k) \times K_2)$ by the rule

$$f(v) = \begin{cases} (v,0) & if \quad |v| = k \\ (v^c,1) & if \quad |v| = n-k, \end{cases}$$

where v^c is the complement of the set v in the set $[n]$. It is an easy task to show that f is a bijection. Let $\{v,w\}$ be an edge in the graph $H(n,k)$, with $|v| = k$, then $v \subset w$, and hence $v \cap w^c = \emptyset$. Therefore v and w^c are adjacent vertices in the Kneser graph $K(n,k)$. Hence $f(v) = (v,0)$ and $f(w) = (w^c,1)$ are adjacent vertices in the graph $K(n,k) \times K_2$. □

It is not difficult to show that the Kneser graph $K(n,k)$ has odd cycles, hence we can deduce by Theorem 3.4, Theorem 3.5 and Theorem 3.6 that the bipartite Kneser graph $H(n,k)$ is a connected graph.

Remark 1. Note that $K_2 = Cay(\mathbb{Z}_2, S)$, where $S = \{1\}$. Therefore, if for some n,k the Kneser graph $K(n,k)$ is a Cayley graph, then by Theorem 3.5. and Theorem 3.6. we conclude that the bipartite Kneser graph $H(n,k)$ is a Cayley graph.

We are now ready to determine for what values of n,k the bipartite Kneser graph $H(n,k)$ is a Cayley graph.

A permutation group G, acting on a set $V(|V| = n)$ is *k-homogeneous* if its induced action on $V^{\{k\}}$ is transitive, where $V^{\{k\}}$ is the set of all k-subsets of V. Also we say that G is *k-transitive* if G is transitive on $V^{(k)}$, where $V^{(k)}$ is the set of k-tuples of distinct elements of V. Note that if G is k-homogeneous, then we have $\binom{n}{k}$ $|G|$ and if G is k-transitive, then we have $\frac{n!}{(n-k)!} \mid |G|$. If the group G acts regularly on $V^{(k)}$, then G is said to be sharply k-transitive on V. This means that for given two k-tuples in $V^{(k)}$, there is a unique permutation in G mapping one k-tuple to the other. In this scope, we have the following result [5. Theorem 9.4B, 9] which is a deep result in group theory.

Theorem 4.4. *Let G be a k-homogeneous group on a finite set Ω, $|\Omega| = n$, where $2 \leq k \leq \frac{n}{2}$. Then G is $(k-1)$-transitive, and with the following exceptions G is m-transitive:*
(a) $k = 4$ and, $G = PGL_2(8)$, $P\Gamma L_2(8)$, $P\Gamma L_2(32)$;
(b) $k = 3$ and, $PSL_2(q) \leq G \leq P\Sigma L_1(q)$, $q \equiv 3 \pmod 4$;
(c) $k = 3$ and, $G = AGL_1(8)$, $A\Gamma L_1(8)$, $A\Gamma L_1(32)$;
(d) $k = 2$ and, $ASL_1(q) \leq G \leq A\Sigma L_1(q)$, $q \equiv 3 \pmod 4$.

Godsil [6], by using the above theorem, proved the following result.

Theorem 4.5. [6] *Except in the following cases, the Kneser graph $K(n,k)$ is not a Cayley graph.*

(1) *$k = 2$, n is a prime-power and $n \equiv 3 \pmod 4$.*
(2) *$k = 2$, $n = 8$ or 32.*

We are now ready to prove the following important result.

Theorem 4.6. *Let $n \geq 5$ and $k \geq 2$. If $\Gamma = H(n,k)$ is a bipartite Kneser graph, then except in the following cases, the graph Γ is not a Cayley graph:*
(1) *$k = 2$, n is a prime-power and $n \equiv 3 \pmod 4$.*
(2) *$k = 2$, $n = 8$ or 32.*

Proof. We know by Theorem 3.2, that $Aut(H(n,k)) = H = \{f_\gamma \alpha^i \mid \gamma \in Sym([n]), 0 \leq i \leq 1\} (\cong Sym([n]) \times \mathbb{Z}_2)$, where α and f_γ are automorphisms of the graph Γ which are defined in Proposition 3.1. Suppose that $\Gamma = H(n,k)$ is a Cayley graph. Then, $Aut(H(n,k))$ has a subgroup R such that R acts regularly on the set $V(H(n,k))$. Then $|R| = |V(\Gamma)| = 2\binom{n}{k} = 2\frac{n!}{(k!)(n-k)!}$. If r is an element of R, then by Theorem 3.7 r has a form such as $f_\sigma \alpha^i$, where $\sigma \in Sym([n])$ and $i \in \{0,1\}$. It is an easy task to show that $\alpha f_\sigma = f_\sigma \alpha$, for every $\sigma \in Sym([n])$[16]. If $f_\sigma \alpha^i \in R$, then

$$(f_\sigma \alpha^i)(f_\sigma \alpha^i) = f_\sigma f_\sigma (\alpha^i)^2 = {f_\sigma}^2 = f_{\sigma^2} \in R.$$

Then there are elements of the form f_θ, $\theta \in Sym([n])$ in R. Let $M_1 = \{f_\phi \mid f_\phi \in R\}$, we can easily see that M_1 is a subgroup of R. Since R acts transitively on $V(H(n,k))$, hence R contains elements of the form $f_\theta \alpha$. We let $M_2 = \{f_\theta \alpha \mid f_\theta \alpha \in R\}$. Let $f_{\theta_0}\alpha$ be a fixed element of M_2. Then $M_2 f_{\theta_0}\alpha \subseteq M_1$, because $(f_\theta \alpha)(f_{\theta_0}\alpha) = f_\theta f_{\theta_0}(\alpha)^2 = f_\theta f_{\theta_0} = f_{\theta\theta_0}$. Then, $|M_2| \leq |M_1|$. Since $M_1 f_{\theta_0}\alpha \subseteq M_2$, $|M_1| \leq |M_2|$, and hence $|M_1| = |M_2| = (1/2)|R| = \binom{n}{k}$. We can see that the group M_1 is transitive on the set $V_1 = \{v \mid v \subset [n], |v| = k\}$. Moreover, we can see that M_1 is a subgroup of the automorphism group of the Kneser graph $K(n,k)$ that acts regularly on its vertex-set. In other words, the Kneser graph $K(n,k)$ is a Cayley graph. Now, by Theorem 3.9 and Remark 1 the result follows. □

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