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Sanskruti index of bridge graph and some nanocones

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Abstract. Sanskruti index is the important topological index used to test the chemical properties of chemical comopounds. In this paper, first we obtain the formulae for calculating the Sanskruti index of bridge graph and carbon nanocones $CNC_n(k)$. In addition, Sanskruti index of the line graph of $CNC_k[n]$ nanocones are obtained.

Keywords: Sanskruti index, bridge graph, carbon nanocones
Mathematics Subject Classification (2010): 05C09.

1 Introduction

Graph theory has provided chemist with a variety of useful tools, such as topological indices. Molecules and molecular compounds are often modeled by molecular graph. A molecular graph is a representation of the structural formula of a chemical compound in terms of graph theory, whose vertices correspond to the atoms of the compound and edges correspond to chemical bonds. A graph G with vertex set $V(G)$ and edge set $E(G)$ is connected, if there exists a connection between any pair of vertices in G. For a graph G, the degree of a vertex v is the number of edges incident to v and denoted by $d_G(v)$.

A graph can be recognized by a numeric number, a polynomial, a sequence of numbers or a matrix which represents the whole graph, and these representations are aimed to be uniquely defined for that graph. A topological index is a numeric quantity associated with a graph which characterize the topology of graph and is invariant under graph automorphism. There are some major classes of topological indices such as distance based topolog-

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ical indices, degree based topological indices and counting related polynomials and indices of graphs. Among these classes degree based topological indices are y and particularly in chemistry. In more precise way, a topological index $Top(G)$ of a graph G, is a number with the property that for every graph H isomorphic to G, $Top(G) = Top(H)$. The concept of topological index came from work done by Wiener [13] while he was working on boiling point of paraffin. He named this index as path number. Later on, the path number was renamed as Wiener index. The Wiener index is the first and most studied topological index, both from theoretical point of view and applications, and defined as the sum of distances between all pairs of vertices in G, see for details [2,7].

One of the well-known degree based topological index is the atom-bond connectivity(ABC) index of G, proposed by Estrada et al. in [4], and defined as

$$ABC(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}}.$$

Inspired by work on the ABC index, Furtula et al. [5] proposed the following modified version of the ABC index and called it as *augmented Zagreb index*(AZI) which is defined as

$$AZI(G) = \sum_{uv\in E(G)} \left(\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)-2}\right)^3.$$

The prediction power is better than the ABC index in the study of heat of formation for heptanes and octanes [5]. Motivated by the previous research on topological descriptors and their applications, Hosamani [12] proposed a new index of a molecular graph G called *Sanskruti index S(G)* which is defined as

$$S(G) = \sum_{uv\in E(G)} \left(\frac{s_G(u)s_G(v)}{s_G(u)+s_G(v)-2}\right)^3,$$

where $s_G(u)$ is the sum of degrees of all vertices adjacent to the vertex u, that is,

$$s_G(u) = \sum_{v\in N_G(u)} d_G(v),$$

where $N_G(u)$ is the set of all neighbors of the given vertex u, that is,

$$N_G(u) = \{v \in V(G) | uv \in E(G)\}.$$

In [12] the chemical applicability of the S-index is given and the value of S-index for line graphs of subdivison graphs of 2D-lattice, nanotube and nanotorus of $TUC_4C_8[p,q]$ are computed. In this paper, we obtain the formulae for calculating the Sanskruti index of bridge graph and carbon nanocones $CNC_n(k)$. In addition, Sanskruti index of the line graph of $CNC_k[n]$ nanocones are obtained.

2 Bridge graph

In this section, we obtain the S-index of bridge graph. Set $T = \{t_1, t_2, \ldots, t_k\} \subseteq V(G)$. Then the truncated Sanskruti index S^T is defined as

$$S^{\{t_1,t_2,\ldots,t_k\}}(G) = S^T(G) = \sum_{\substack{uv \in E(G) \\ u,v \notin T}} \Big(\frac{s_G(u)s_G(v)}{s_G(u) + s_G(v) - 2} \Big)^3.$$

If T is empty, then $S^T(G) = S(G)$.

Let G_i, $i \in \{1,2,\ldots,n\}$ be certain graphs and $v_i \in V(G_i)$. The *bridge graph* expressed by $G = G(G_1, G_2, \ldots, G_n, v_1, v_2, \ldots, v_n)$ which can be regarded as the union of the graphs G_i, $i \in \{1,2,\ldots,n\}$ via connected edges v_iv_{i+1}, $i \in \{1,2,\ldots,n-1\}$, see Figure 1. One can see that the number of vertices and edges of the bridge graph are $\sum\limits_{i=1}^{n} |V(G_i)|$ and $\sum\limits_{i=1}^{n} |E(G_i)| + (n-1)$, respectively.

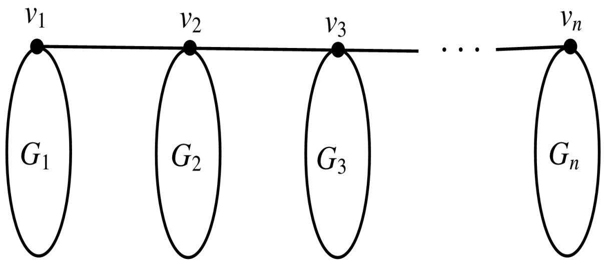

Figure 1. The bridge graph $G = G(G_1, \ldots, G_n, v_1, \ldots, v_n)$.

The following lemma is easily obtained from the structure of bridge graph.

Lemma 2.1. *Let $G = G(G_1, G_2, \ldots, G_n, v_1, v_2, \ldots, v_n)$ be a bridge graph and $N_G[u] = N_G(u) \cup \{u\}$ for each $u \in V(G)$. Then the following is true:*

(i) $G = G(G_1, G_2, \ldots, G_n, v_1, v_2, \ldots, v_n)$ is connected if and only if G_i, $i \in \{1,2,\ldots,n\}$ are connected.

(ii) The degree of a vertex $v \in V(G)$ is

$$d_G(v) = \begin{cases} d_{G_i}(v), & v \in V(G_i) \text{ and } v \neq v_i \\ d_{G_i}(v) + 1, & v = v_i \text{ and } i \in \{1,n\} \\ d_{G_i}(v) + 2, & v = v_i \text{ and } i \in \{2,\ldots,n-1\}. \end{cases}$$

(iii) If $u \in V(G_i)$ and $v_i \notin N_{G_i}[u]$, then $s_G(u) = s_{G_i}(u)$ where $s_G(u) = s(u)$ and

$$s_{G_i}(u) = \sum_{v \in N_{G_i}(u)} d_{G_i}(v).$$

Theorem 2.2. *Set $T = \{t_1, t_2, \ldots, t_k\} \subseteq V(G)$ and suppose $v_1, v_2, \ldots, v_n \notin T$. For bridge graph $G = G(G_1, G_2, \ldots, G_n, v_1, v_2, \ldots, v_n)$, we have*

$$S^{\{t_1,t_2,\ldots,t_k\}}(G) = \sum_{i=1}^{n} S^{T\cup N_{G_i}[v_i]}(G_i) + \sum_{i=1}^{n} \sum_{\substack{uv\in E(G_i)\\ u,v\notin T,\ u\in N_{G_i}[v_i]}} \Big(\frac{s(u)s(v)}{s(u)+s(v)-2}\Big)^3 + \sum_{i=1}^{n-1} \Big(\frac{s(v)s(v_{i+1})}{s(v_i)+s(v_{i+1})-2}\Big)^3.$$

Proof. By the definition of truncated S-index,

$$\begin{aligned}
S^{\{t_1,t_2,\ldots,t_k\}}(G) &= \sum_{\substack{uv\in E(G)\\ u,v\notin T}} \Big(\frac{s(u)s(v)}{s(u)+s(v)-2}\Big)^3 + \sum_{i=1}^{n} \sum_{\substack{uv\in E(G_i)\\ u,v\notin T}} \Big(\frac{s(u)s(v)}{s(u)+s(v)-2}\Big)^3 \\
&\quad + \sum_{i=1}^{n-1} \Big(\frac{s(v)s(v_{i+1})}{s(v_i)+s(v_{i+1})-2}\Big)^3 \\
&= \sum_{i=1}^{n} \sum_{\substack{uv\in E(G_i)\\ u,v\notin T\cup N_{G_i}[v_i]}} \Big(\frac{s_{G_i}(u)s_{G_i}(v)}{s_{G_i}(u)+s_{G_i}(v)-2}\Big)^3 \\
&\quad + \sum_{i=1}^{n} \sum_{\substack{uv\in E(G_i)\\ u,v\notin T,\ u\in N_{G_i}[v_i]}} \Big(\frac{s(u)s(v)}{s(u)+s(v)-2}\Big)^3 + \sum_{i=1}^{n-1} \Big(\frac{s(v)s(v_{i+1})}{s(v_i)+s(v_{i+1})-2}\Big)^3 \\
&= \sum_{i=1}^{n} S^{T\cup N_{G_i}[v_i]}(G_i) + \sum_{i=1}^{n} \sum_{\substack{uv\in E(G_i)\\ u,v\notin T,\ u\in N_{G_i}[v_i]}} \Big(\frac{s(u)s(v)}{s(u)+s(v)-2}\Big)^3 \\
&\quad + \sum_{i=1}^{n-1} \Big(\frac{s(v)s(v_{i+1})}{s(v_i)+s(v_{i+1})-2}\Big)^3.
\end{aligned}$$

□

By setting $n = 2$ in above Theorem, we obtain the following corollary.

Corollary 2.3. *For $G = G(G_1, G_2, v_1, v_2)(T = \{t_1, t_2, \ldots, t_k\} \subseteq V(G), v_1, v_2 \notin T)$, we have*

$$\begin{aligned}
S^{\{t_1,t_2,\ldots,t_k\}}(G) &= \sum_{i=1}^{2} S^{T\cup N_{G_i}[v_i]}(G_i) + \sum_{i=1}^{2} \sum_{\substack{uv\in E(G_i)\\ u,v\notin T,\ u\in N_{G_i}[v_i]}} \Big(\frac{s(u)s(v)}{s(u)+s(v)-2}\Big)^3 \\
&\quad + \Big(\frac{(s_{G_1}(v_1)+d_{G_2}(v_2)+1)(s_{G_2}(v_2)+d_{G_1}(v_1)+1)}{(s_{G_1}(v_1)+s_{G_2}(v_2)+d_{G_1}(v_1)+d_{G_2}(v_2))}\Big)^3.
\end{aligned}$$

Using above corollary, we obtain the S-index of some special molecular graphs.

Example 2.4. Consider the nanostar G_1 expressed in Figure 2. One can check that

$$\begin{aligned}S(G_1) &= 6\Big(\frac{4(4)}{4+4-2}\Big)^3 + 3\Big(\frac{7(9)}{7+9-2}\Big)^3 + 6\Big(\frac{4(5)}{4+5-2}\Big)^3 + 6\Big(\frac{5(7)}{5+7-2}\Big)^3 \\ &= 6\Big(\frac{8}{3}\Big)^3 + 3\Big(\frac{9}{2}\Big)^3 + 6\Big(\frac{20}{7}\Big)^3 + 6\Big(\frac{7}{2}\Big)^3.\end{aligned}$$

Further,

$$\begin{aligned}S^{N_{G_1}[v_1]}(G_1) = S^{N_{G_1}[v_2]}(G_1) &= S^{N_{G_1}[v_3]}(G_1) \\ &= 4\Big(\frac{8}{3}\Big)^3 + 3\Big(\frac{9}{2}\Big)^3 + 4\Big(\frac{20}{7}\Big)^3 + 6\Big(\frac{7}{2}\Big)^3.\end{aligned}$$

and for any $1 \leq i, j \leq 3$ and $i \neq j$ we have

$$S^{N_{G_1}[v_i] \cup N_{G_1}[v_j]}(G_1) = 2\Big(\frac{8}{3}\Big)^3 + 3\Big(\frac{9}{2}\Big)^3 + 2\Big(\frac{20}{7}\Big)^3 + 6\Big(\frac{7}{2}\Big)^3.$$

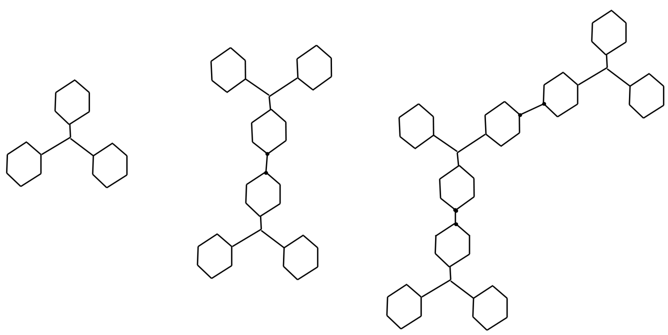

Figure 2. The graph of nanostar dendrimer D_n for $n = 1,2,3$.

Now consider the bridge graph $G_n = G(G_{n-1}, H_1, v_1, t_1)$ manifested in Figure 2. Observe that $H_i \cong G_1$ for $i \in \{1,2,\ldots,n-1\}$ and

$$\begin{aligned}G_n &= G(G_{n-1}, H_1, v_1, t_1), \\ G_{n-1} &= G(G_{n-2}, H_2, v_2, t_2), \\ &\vdots \\ G_{n-i} &= G(G_{n-i-1}, H_{i+1}, v_{i+1}, t_{i+1}), \\ &\vdots \\ G_2 &= G(G_1, H_{n-1}, v_{n-1}, t_{n-1}).\end{aligned}$$

Hence, by Corollary 2.3, we get the following relationships:

$$\begin{aligned}
S(G_n) &= S^{N_{G_{n-1}}[v_1]}(G_{n-1}) + S^{N_{H_1}[t_1]}(H_1) + r,\\
S^{N_{G_{n-1}}[v_1]}(G_{n-1}) &= S^{N_{G_{n-2}}[v_2]}(G_{n-2}) + S^{N_{H_2}[v_1]\cup N_{H_2}[t_2]}(H_2) + r,\\
&\vdots\\
S^{N_{G_{n-i}}[v_i]}(G_{n-i}) &= S^{N_{G_{n-i-1}}[v_{i+1}]}(G_{n-i-1}) + S^{N_{H_{i+1}}[v_i]\cup N_{H_{i+1}}[t_{i+1}]}(H_{i+1}) + r,\\
&\vdots\\
S^{N_{G_2}[v_{n-2}]}(G_2) &= S^{N_{G_1}[v_{n-1}]}(G_1) + S^{N_{H_{n-1}}[v_{n-2}]\cup N_{H_{n-1}}[t_{n-1}]}(H_{n-1}) + r,
\end{aligned}$$

where

$$\begin{aligned}
r &= \Big(\frac{7(7)}{7+7-2}\Big)^3 + 4\Big(\frac{5(5)}{5+5-2}\Big)^3 + 4\Big(\frac{5(7)}{5+7-2}\Big)^3\\
&= \Big(\frac{49}{12}\Big)^3 + 4\Big(\frac{25}{8}\Big)^3 + 4\Big(\frac{7}{2}\Big)^3.
\end{aligned}$$

Combining those relationship stated above, we have

$$S(G_n) = S^{N_{G_1}[v_{n-1}]}(G_1) + S^{N_{H_1}[t_1]}(H_1) + \sum_{i=2}^{n-1} S^{N_{H_i}[v_{i-1}]\cup N_{H_i}[t_i]}(H_i) + (n-1)r.$$

Therefore,

$$\begin{aligned}
S(G_n) &= 2S^{N_{G_1}[v_1]}(G_1) + (n-2)S^{G_1[v_1]\cup N_{G_1}[v_2]}(G_1) + (n-1)r\\
&= 2\Big[4\Big(\frac{8}{3}\Big)^3 + 3\Big(\frac{9}{2}\Big)^3 + 4\Big(\frac{20}{7}\Big)^3 + 6\Big(\frac{7}{2}\Big)^3\Big]\\
&\quad +(n-2)\Big[2\Big(\frac{8}{3}\Big)^3 + 3\Big(\frac{9}{2}\Big)^3 + 2\Big(\frac{20}{7}\Big)^3 + 6\Big(\frac{7}{2}\Big)^3\Big]\\
&\quad +\Big[\Big(\frac{49}{12}\Big)^3 + 4\Big(\frac{25}{8}\Big)^3 + 4\Big(\frac{7}{2}\Big)^3\Big]\\
&= \Big(273.375\Big)n + (85.75)(5n-2) + 2(n+2)\Big[\Big(\frac{8}{3}\Big)^3 + \Big(\frac{20}{7}\Big)^3\Big]\\
&\quad +(n-1)\Big[\Big(\frac{49}{12}\Big)^3 + \Big(\frac{25}{8}\Big)^3\Big].
\end{aligned}$$

Using above, we have the following theorems.

Theorem 2.5. *Let $G_n = G(G_{n-1}, H_1, v_1, t_1)$ be the bridge graph presented in Figure 2. Then*

$$\begin{aligned}
S(G_n) &= \Big(273.375\Big)n + (85.75)(5n-2) + 2(n+2)\Big[\Big(\frac{8}{3}\Big)^3 + \Big(\frac{20}{7}\Big)^3\Big]\\
&\quad +(n-1)\Big[\Big(\frac{49}{12}\Big)^3 + \Big(\frac{25}{8}\Big)^3\Big].
\end{aligned}$$

Theorem 2.6. *Let D be the nanostar dendrimer. Then*

$$\begin{aligned}
S(D) &= \Big(273.375\Big)n + (85.75)(5n-2) + 2(n+2)\Big[\Big(\frac{8}{3}\Big)^3 + \Big(\frac{20}{7}\Big)^3\Big]\\
&\quad +(n-1)\Big[\Big(\frac{49}{12}\Big)^3 + \Big(\frac{25}{8}\Big)^3\Big].
\end{aligned}$$

3 Carbon nanocones

In this section, we compute the S-index of famous carbon nanocones $CNC_n(k)$. One can see that the number of vertices of $CNC_n(k)$ is $n(k+1)^2$ and the number of edges of $CNC_n(k)$ is $\frac{n}{2}(k+1)(3k+2)$. Before presenting our main result in this section, we first see the following two examples.

Example 3.1. Consider the carbon nanocones $CNC_3(1)$ shown in Figure 3. This molecular structure has 15 edges, where three of them with $s(u)=s(v)=5$, six of them with $s(u)=7$ and $s(v)=5$, three of them with $s(u)=7$ and $s(v)=9$, and three of them with $s(u)=s(v)=9$. From the definition of S-index, we have

$$S(CNC_3(1)) = 3\Big(\frac{5(5)}{5+5-2}\Big)^3 + 6\Big(\frac{5(7)}{5+7-2}\Big)^3 + 3\Big(\frac{7(9)}{7+9-2}\Big)^3 + 3\Big(\frac{9(9)}{9+9-2}\Big)^3$$
$$= (530.625) + \Big(\frac{46875}{512}\Big) + 3\Big(\frac{531441}{4096}\Big).$$

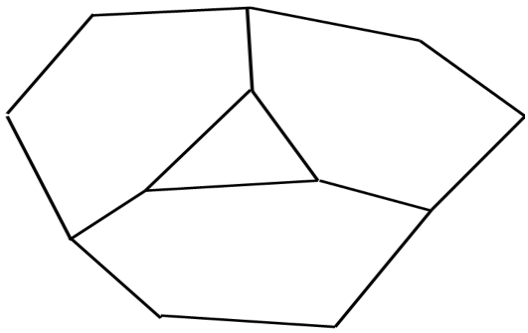

Figure 3. The carbon nanocones $CNC_3(1)$.

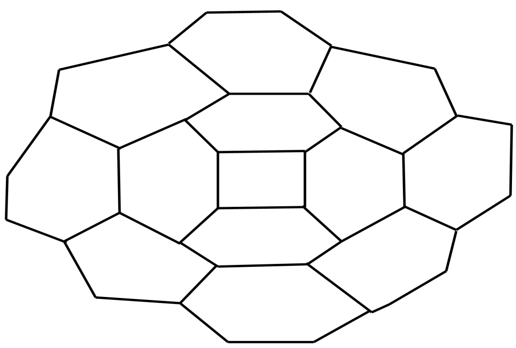

Figure 4. The carbon nanocones $CNC_4(2)$.

Example 3.2. Consider the carbon nanocones $CNC_4(2)$ shown in Figure 4. This molecular structure has 48 edges, where four of them with $s(u)=s(v)=5$, eight of them with $s(u)=7$

and $s(v)=5$, eight of them with $s(u)=7$ and $s(v)=9$, eight of them with $s(u)=7$ and $s(v)=6$, and twenty of them with $s(u)=s(v)=9$. Then

$$\begin{aligned}S(CNC_4(2)) &= 4\Big(\frac{5(5)}{5+5-2}\Big)^3+8\Big(\frac{5(7)}{5+7-2}\Big)^3+8\Big(\frac{7(9)}{7+9-2}\Big)^3\\ &+20\Big(\frac{9(9)}{9+9-2}\Big)^3+8\Big(\frac{7(6)}{7+6-2}\Big)^3\\ &=(1072)+\Big(\frac{15625}{128}\Big)+8\Big(\frac{74088}{1331}\Big)+20\Big(\frac{531441}{4096}\Big).\end{aligned}$$

Now we obtain the main result for this section.

Theorem 3.3. *Let $n\geq 2$ and $k\geq 1$ be positive integers. Then*

$$S(CNC_n(k))=\frac{4nk(3k-1)}{9}+\frac{n(5k+6)}{15}\sqrt{2}+2n\sqrt{\frac{2}{7}}+2n(k-1)\sqrt{\frac{11}{42}}.$$

Proof. From the Examples 2 and 3, we obtain

$$\begin{aligned}S(CNC_n(k)) &= n\Big(\frac{5(5)}{5+5-2}\Big)^3+2n\Big(\frac{5(7)}{5+7-2}\Big)^3+nk\Big(\frac{7(9)}{7+9-2}\Big)^3\\ &+2(k-1)n\Big(\frac{7(6)}{7+6-2}\Big)^3+\frac{nk(3k-1)}{2}\Big(\frac{9(9)}{9+9-2}\Big)^3\\ &=n\Big(\frac{3366987}{681472}\Big)+nk\Big(\frac{269463.375}{1331}\Big)+nk(3k-1)\Big(\frac{531441}{8192}\Big).\end{aligned}$$

□

4 Line graph of $CNC_k[n]$ nanocones

In this section, we find the S-index of line graph of $CNC_k[n]$ nanocones. The following lemma is useful to finding the degree of a vertex of a line graph. Let G be a graph, $u\in V(G)$ and $e=uv\in E(G)$. Then $d(e)=d(u)+d(v)-2$.

Example 4.1. Consider the line graph of $CNC_3[1]$. In this graph we have 6 edges of $s(u)=6$ and $s(v)=9$, 3 edges of $s(u)=s(v)=9$, 6 edges of $s(u)=9$ and $s(v)=14$, 6 edges of $s(u)=14$ and $s(v)=16$ and 3 edges of $s(u)=s(v)=16$. Thus

$$S(L(CNC_3[1]))=4368+\frac{6(54)^3}{2197}+\frac{3(81)^3}{4096}+\frac{(128)^3}{1125}.$$

Theorem 4.2. *Let G be a line graph of $CNC_3[n]$ nanocones for $n>1$. Then*

$$\begin{aligned}S(L(CNC_3[n])) &= n^2\Big(\frac{(128)^3}{375}\Big)+n\Big[3072+\frac{2(50)^3}{243}+\frac{6(70)^3}{1331}-\frac{2(128)^3}{1125}\Big]\\ &+6\Big[\frac{(54)^3}{2197}+\frac{(90)^3}{4913}+\frac{(42)^3}{343}-\frac{(70)^3}{1331}\Big].\end{aligned}$$

Proof. The graph G consists of $3(\frac{3}{2}n^2+\frac{5}{2}n+1)$ vertices and $3(n+1)(3n+1)$ edges. There are seven types of edges in $E(G)$ based on the degree sum of vertices lying at the unit distance from end vertices of each edge. The edge partition E_1 contains 6 edges where $s(u)=6$ and $s(v)=9$, the edge partition E_2 contains 6 edges $s(u)=9$ and $s(v)=10$, the edge partition E_3 contains 6 edges where $s(u)=9$ and $s(v)=14$, the edge partition E_4 contains $6n-9$ edges where $s(u)=10$ and $s(v)=10$, the edge partition E_5 contains $6n-6$ edges where $s(u)=10$ and $s(v)=14$, the edge partition E_6 contains $6n$ edges where $s(u)=14$ and $s(v)=16$ and the edge partition E_7 contains $9n^2-6n$ edges where $s(u)=s(v)=16$.

$$\begin{aligned}
S(G) &= \sum_{uv\in E(G)}\Big(\frac{s(u)s(v)}{s(u)+s(v)-2}\Big)^3\\
&= 6\Big(\frac{6(9)}{6+9-2}\Big)^3+6\Big(\frac{9(10)}{9+10-2}\Big)^3+6\Big(\frac{9(14)}{9+14-2}\Big)^3+(6n-9)\Big(\frac{10(10)}{10+10-2}\Big)^3\\
&\quad+(6n-6)\Big(\frac{10(14)}{10+14-2}\Big)^3+6n\Big(\frac{14(16)}{14+16-2}\Big)^3+(9n^2-6n)\Big(\frac{16(16)}{16+16-2}\Big)^3\\
&= n^2\Big(\frac{(128)^3}{375}\Big)+n\Big[3072+\frac{2(50)^3}{243}+\frac{6(70)^3}{1331}-\frac{2(128)^3}{1125}\Big]\\
&\quad+6\Big[\frac{(54)^3}{2197}+\frac{(90)^3}{4913}+\frac{(42)^3}{343}-\frac{(70)^3}{1331}\Big].
\end{aligned}$$

□

Example 4.3. Consider the line graph of $CNC_4[1]$. In this graph we have 8 edges of $s(u)=6$ and $s(v)=9$, 3 edges of $s(u)=s(v)=9$, 8 edges of $s(u)=9$ and $s(v)=14$, 8 edges of $s(u)=14$ and $s(v)=16$ and 4 edges of $s(u)=s(v)=16$. Thus

$$S(L(CNC_4[1])) = 5824+\frac{8(54)^3}{2197}+\frac{(81)^3}{1024}+\frac{4(128)^3}{3375}.$$

Theorem 4.4. *Let G be a line graph of $CNC_4[n]$ nanocones for $n>1$. Then*

$$\begin{aligned}
S(L(CNC_4[n])) &= 4n^2\Big(\frac{(128)^3}{1125}\Big)+8n\Big[512+\frac{(50)^3}{729}+\frac{(70)^3}{1331}-\frac{(128)^3}{3375}\Big]\\
&\quad+8\Big[1728+\frac{(54)^3}{2197}+\frac{(90)^3}{4913}-\frac{(50)^3}{486}-\frac{(70)^3}{1331}\Big].
\end{aligned}$$

Proof. The graph G consists of $4(\frac{3}{2}n^2+\frac{5}{2}n+1)$ vertices and $4(n+1)(3n+1)$ edges. There are seven types of edges in $E(G)$ based on the degree sum of vertices lying at the unit distance from end vertices of each edge. The edge partition E_1 contains 8 edges where $s(u)=6$ and $s(v)=9$, the edge partition E_2 contains 8 edges $s(u)=9$ and $s(v)=10$, the edge partition E_3 contains 8 edges where $s(u)=9$ and $s(v)=14$, the edge partition E_4 contains $8n-12$ edges where $s(u)=s(v)=10$, the edge partition E_5 contains $8n-8$ edges where $s(u)=10$ and $s(v)=14$, the edge partition E_6 contains $8n$ edges where $s(u)=14$ and $s(v)=16$ and the edge partition E_7 contains $12n^2 - 8n$ edges where $s(u) = s(v) = 16$. Thus

$$\begin{aligned}
S(G) &= \sum_{uv \in E(G)} \Big(\frac{s(u)s(v)}{s(u)+s(v)-2} \Big)^3 \\
&= 8\Big(\frac{6(9)}{6+9-2}\Big)^3 + 8\Big(\frac{9(10)}{9+10-2}\Big)^3 + 8\Big(\frac{9(14)}{9+14-2}\Big)^3 + (8n-12)\Big(\frac{10(10)}{10+10-2}\Big)^3 \\
&\quad + (8n-8)\Big(\frac{10(14)}{10+14-2}\Big)^3 + 8n\Big(\frac{14(16)}{14+16-2}\Big)^3 + (12n^2-8n)\Big(\frac{16(16)}{16+16-2}\Big)^3 \\
&= 4n^2\Big(\frac{(128)^3}{1125}\Big) + 8n\Big[512 + \frac{(50)^3}{729} + \frac{(70)^3}{1331} - \frac{(128)^3}{3375}\Big] \\
&\quad + 8\Big[1728 + \frac{(54)^3}{2197} + \frac{(90)^3}{4913} - \frac{(50)^3}{486} - \frac{(70)^3}{1331}\Big].
\end{aligned}$$

□

Example 4.5. Consider the line graph of $CNC_k[1]$. In this graph we have $2k$ edges of $s(u) = 6$ and $s(v) = 9$, k edges of $s(u) = s(v) = 9$, $2k$ edges of $s(u) = 9$ and $s(v) = 14$, $2k$ edges of $s(u) = 14$ and $s(v) = 16$ and k edges of $s(u) = s(v) = 16$. Thus

$$S(L(CNC_k[1])) = k\Big[\frac{3513760}{2197} + \frac{531441}{4096} + \frac{2097152}{3375}\Big].$$

Theorem 4.6. *Let G be a line graph of $CNC_k[n]$ nanocones for $n > 1$. Then*

$$\begin{aligned}
S(L(CNC_k[n])) &= kn^2\Big(\frac{(128)^3}{1125}\Big) + 2kn\Big[512 + \frac{(50)^3}{729} + \frac{(70)^3}{1331} - \frac{(128)^3}{3375}\Big] \\
&\quad + k\Big[432 + \frac{(54)^3}{2197} + \frac{2(90)^3}{4913} - \frac{(50)^3}{243} - \frac{2(70)^3}{1331}\Big].
\end{aligned}$$

Proof. The graph G consists of $2k(\frac{3}{2}n^2 + \frac{5}{2}n + 1)$ vertices and $k(n+1)(3n+1)$ edges. There are seven types of edges in $E(G)$ based on the degree sum of vertices lying at the unit distance from end vertices of each edge. The edge partition E_1 contains $2k$ edges where $s(u) = 6$ and $s(v) = 9$, the edge partition E_2 contains $2k$ edges $s(u) = 9$ and $s(v) = 10$, the edge partition E_3 contains $2k$ edges where $s(u) = 9$ and $s(v) = 14$, the edge partition E_4 contains $2kn - 3k$ edges where $s(u) = s(v) = 10$, the edge partition E_5 contains $2kn - 2k$ edges where $s(u) = 10$ and $s(v) = 14$, the edge partition E_6 contains $2kn$ edges where $s(u) = 14$ and $s(v) = 16$ and the edge partition E_7 contains $3kn^2 - 2kn$ edges where $s(u) = s(v) = 16$. Thus

$$
\begin{aligned}
S(G) &= \sum_{uv \in E(G)} \left(\frac{s(u)s(v)}{s(u)+s(v)-2} \right)^3 \\
&= 2k\left(\frac{6(9)}{6+9-2}\right)^3 + 2k\left(\frac{9(10)}{9+10-2}\right)^3 + 2k\left(\frac{9(14)}{9+14-2}\right)^3 + (2kn-3k)\left(\frac{10(10)}{10+10-2}\right)^3 \\
&\quad + (2kn-2k)\left(\frac{10(14)}{10+14-2}\right)^3 + 2kn\left(\frac{14(16)}{14+16-2}\right)^3 + (3kn^2-2kn)\left(\frac{16(16)}{16+16-2}\right)^3 \\
&= kn^2\left(\frac{(128)^3}{1125}\right) + 2kn\left[512 + \frac{(50)^3}{729} + \frac{(70)^3}{1331} - \frac{(128)^3}{3375}\right] \\
&\quad + k\left[432 + \frac{(54)^3}{2197} + \frac{2(90)^3}{4913} - \frac{(50)^3}{243} - \frac{2(70)^3}{1331}\right].
\end{aligned}
$$

□

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