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*Research Paper*

# Finite groups whose enhanced power graphs are unique

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**Abstract.** The enhanced power graph $P_e(G)$ of a group $G$ is the graph whose vertex set is $G$, with two elements $u$ and $v$ adjacent if there is an element $z \in G$ such that $\langle u, v \rangle = \langle z \rangle$. In this paper, we investigate classes of groups whose enhanced power graphs uniquely determine their structure; that is, if $P_e(G) \cong P_e(H)$, then $G \cong H$. We also study the set of natural numbers $n$ for which every group of order $n$ is uniquely determined (up to isomorphism) by its enhanced power graph. We consider groups that have the same number of elements of each order and exploit necessary conditions to identify situations in which a property of a group $G$ is preserved by all groups sharing the same enhanced power graph. In particular, we show that if two finite groups have isomorphic enhanced power graphs and one of them is nilpotent or has a normal Hall subgroup, then the other must also share that property.

**Keywords.** enhanced power graph, power graph, nilpotent group, normal Hall subgroup.
**Mathematics Subject Classification (2020):** 05$C$12, 91$A$43, 05$C$69.

## 1 Introduction

The **power graph** of a group $G$, $Pow(G)$, is a graph where the vertex set is $G$, and two distinct elements $x$ and $y$ are adjacent if one is a power of the other.

The **commuting graph** of a group $G$, is a graph where the vertex set is $G$, and two distinct elements $x$ and $y$ are adjacent if they commute, meaning $xy = yx$.

The **enhanced power graph** of a group $G$, denoted by $P_e(G)$ is defined as a simple graph with vertex set $G$. Two distinct vertices $x$ and $y$ are adjacent if and only if $\langle x, y \rangle$, the subgroup generated by $x$ and $y$ is cyclic.

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The concept of the enhanced power graph was formally introduced by Aalipour in [1] as a graph that lies between the power graph and the commuting graph of a group. However, the idea can be traced back to the work of Xuanlong Ma in [6], who introduced the **cyclic graph** of a finite group.

The commuting graph is the complement of the non-commuting graph, a concept initially explored by Erdos [9]. Similarly, the enhanced power graph is closely related to the complement of the non-cyclic graph (which we refer to as the cyclic graph), with a difference in the vertex set. The non-cyclic graph of a group, introduced in [2,3], as the graph with two vertices $x$ and $y$ are adjacent if the subgroup $\langle x,y \rangle$ is non-cyclic. In this definition, isolated vertices are excluded.

It can be observed that for any group $G$, the graph $Pow(G)$ is a spanning subgraph of $P_e(G)$. The following result characterizes the equality of these graphs:

For any finite group $G$, $Pow(G) \cong P_e(G)$ if and only if every cyclic subgroup of $G$ has prime power order, [1]. Therefore, for any finite $p$-group $G$, the graphs $Pow(G)$ and $P_e(G)$ are equal.

We are interested in whether an isomorphism between the enhanced power graphs of two groups implies an isomorphism between the groups themselves. In other words, if $P_e(G_1) \cong P_e(G_2)$, does it follow that $G_1 \cong G_2$? This is not true in general. For example, consider the nilpotent groups $\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3$ and the group $S$ given by the presentation

$$S = \langle x,y \mid x^3 = y^3 = [x,y]^3 = 1 \rangle,$$

where $[x,y]$ denotes the commutator $xyx^{-1}y^{-1}$. Although these groups are not isomorphic, their enhanced power graphs are isomorphic.

Corollary 3.1 in [10] establishes that two groups have isomorphic power graphs if and only if their enhanced power graphs are isomorphic. It was shown in [4] that if $\mathrm{Pow}(G) \cong \mathrm{Pow}(H)$, then $G$ and $H$ have the same number of elements of each order. Moreover, abelian groups with isomorphic enhanced power graphs must also be isomorphic. These two results also hold for enhanced power graphs.

In [8], it is proven that if $G$ is one of the following finite groups: a cyclic group, a symmetric group, a dihedral group, or a generalized quaternion group, and $H$ is a finite group such that $Pow(G) \cong Pow(H)$, then $G \cong H$. Following this pattern, the same results also holds for the enhanced power graphs too. These studies motivated us to extend our investigation to enhanced power graphs

## 2 Preliminary results

In this section, we introduce the notation and present preliminary results necessary for proving our main theorems.

The set of all element orders of a group $G$, called its spectrum and denoted by $\pi_e(G)$. The set of all element orders of a group $G$, known as its spectrum, is denoted by $\pi_e(G)$. Note that two groups having the same number of elements of each order need not be isomorphic. For example, if $p$ is an odd prime and $m > 2$, then in addition to the elementary abelian group $H$

of order $p^m$, there exist non-abelian groups $G$ of the same order and exponent $p$. Thus, $H$ and $G$ are non-isomorphic, while they have the same number of elements of each order.

The relevance of this concept to power graphs is due to the fact that, as proved by Cameron [5], two finite groups with isomorphic power graphs have the same number of elements of each order. Note that the converse is not true. For example, two groups of order 16 with the same numbers of elements of each order, e.g. $C_4 \times C_4$ and $C_2 \times Q_8$ are SmallGroup(16,2) and SmallGroup(16,4) in GAP respectively. Their power graphs are not isomorphic. In fact, in the group $C_4 \times C_4$, each element of order 2 has four square roots, but in $C_2 \times Q_8$, the involution in $Q_8$ has twelve square roots and the other two have none.

In [15], the authors investigated the following problem: for which natural numbers $n$ is it true that every two groups of order $n$ having the same number of elements of each order are necessarily isomorphic? Denote the set of all such numbers by $S$.

In this paper, we consider a related question: for which natural numbers $n$ does the condition that two groups of order $n$ have isomorphic enhanced power graphs imply that the groups themselves are isomorphic? Let us denote the set of all such numbers by $\bar{S}$.

**Theorem 2.1.** *( [4]) Two finite groups whose power graphs are isomorphic have the same numbers of elements of each order.*

**Theorem 2.2.** *( [10]) For a pair of finite groups, the following are equivalent:*

- *the power graphs are isomorphic;*
- *the directed power graphs are isomorphic;*
- *the enhanced power graphs are isomorphic.*

By Theorem 2.1 and Theorem 2.2, two finite groups with isomorphic enhanced power graphs have the same numbers of elements of each order , it is easy to see that $S \subseteq \bar{S}$.

**Theorem 2.3.** *( [5]) Let $G$ be a finite group; let $S$ be the set of vertices of the power graph $P(G)$ which are joined to all other vertices. Suppose that $|S| > 1$. Then one of the following occurs:*

1. *$G$ is cyclic of prime power order, and $S = G$;*
2. *$G$ is cyclic of non-prime-power order $n$, and $S$ consists of the identity and the generators of $G$, so that $|S| = 1 + \phi(n)$;*
3. *$G$ is generalised quaternion, and $S$ contains the identity and the unique involution in $G$, so that $|S| = 2$.*

This paper is structured as follows:

In Section 3.1, we begin by examining examples of non-isomorphic groups that share the same enhanced power graphs. In Section 3.2, we investigate the group orders that are uniquely determined by their enhanced power graphs. In particular, we address the question: For which $n \in \mathbb{N}$ does the condition that all pairs of groups $G$ and $H$ of order $|G| = |H| = n$, having the same number of elements of each order, imply that $G \cong H$? In Section 3.3, we study enhanced power graphs of nilpotent groups and groups possessing a normal Hall subgroup.

## 3 Main Results

### 3.1 Groups Determined by Their Enhanced Power Graphs

In this section, we investigate finite groups that are uniquely determined (up to isomorphism) by their enhanced power graphs. That is, for such a group $G$, if another group $H$ satisfies $P_e(G) \cong P_e(H)$, then it must follow that $G \cong H$. We present several examples of such groups and prove that their enhanced power graphs characterize the group structure uniquely.

**Remark 3.1.** *By the definition of the enhanced power graph, it is evident that the enhanced power graph of a group $G$ is complete if and only if $G$ is cyclic.*

As a consequence, we conclude that $P_e(G) \cong P_e(\mathbb{Z}_n)$ if and only if $G \cong \mathbb{Z}_n$.

Now we consider the dihedral group of order $2n$ and denote it by $D_{2n}$. The dihedral group $D_{2n}$ is defined by the following

$$D_{2n} = \langle x, y \mid x^n = y^2 = e, y^{-1}xy = x^{-1} \rangle.$$

**Theorem 3.2.** *If $G$ is a group such that $P_e(G) \cong P_e(D_{2n})$, then $G \cong D_{2n}$.*

*Proof.* Assume $P_e(G) \cong P_e(D_{2n})$. Then $|G| = 2n$, and $G$ contains an element $a$ of order $n$. Since $G$ has the same number of elements of order 2 as the dihedral group $D_{2n}$, there exists an element $b \in G$ of order 2 such that $\langle a \rangle \cap \langle b \rangle = \{1\}$. This implies that $G$ is a semidirect product of the cyclic group $\mathbb{Z}_n$ by $\mathbb{Z}_2$, with $b$ acting on $a$ by inversion. Therefore, $G \cong D_{2n}$. □

**Remark 3.3.** *Wujie Shi, conjectured that if $M$ is a finite nonabelian simple group and $G$ is a finite group such that $|G| = |M|$ and $\pi_e(G) = \pi_e(M)$, then $G \cong M$, [11]. This conjecture has been verified for many nonabelian finite simple groups. For instance, if $G$ is a finite group and $M$ is one of the following finite simple groups:*

1. *an alternating group $A_n$, with $n \geq 5$;*
2. *a sporadic simple group;*
3. *a Lie-type group, except for $B_n$, $C_n$, and $D_n$ with even $n$;*
4. *a simple group of order less than $10^8$,*

*then $G \cong M$ if and only if $|G| = |M|$ and $\pi_e(G) = \pi_e(M)$, [12–14]. Moreover, this conjecture has also been confirmed for certain non-simple groups, such as the symmetric group $S_n$ for $n \geq 3$, [**?**].*

It was shown in [2] that the following theorem holds for non-cyclic graphs. We now consider its analogue for enhanced power graphs.

**Theorem 3.4.** *Let $G$ be a group.*

1. *If $n > 2$ and $P_e(G) \cong P_e(S_n)$, then $G \cong S_n$.*

2. *If $n > 3$ and $P_e(G) \cong P_e(A_n)$, then $G \cong A_n$.*

*Proof.* (1) Assume that $P_e(G) \cong P_e(S_n)$. Then $|G| = |S_n|$, and $G$ and $S_n$ have the same number of elements of each order. Therefore, $\pi_e(G) = \pi_e(S_n)$, and it follows that $G \cong S_n$.

(2) If $P_e(G) \cong P_e(A_n)$, then $|G| = |A_n|$ and $\pi_e(G) = \pi_e(A_n)$, which Remark 3.3, implies that $G \cong A_n$ for $n \geq 5$. For the case $n = 4$, note that $A_4$ is the only group of order 12 that has no element of order 6. Therefore, $G \cong A_4$ in this case as well. This completes the proof. □

For $n \geq 2$, the generalized quaternion group $Q_{4n}$ of order $4n$ is given by

$$Q_{4n} = \langle x, y \mid x^n = y^2, x^{2n} = e, y^{-1}xy = x^{-1} \rangle$$

**Theorem 3.5.** *If G is a group with $P_e(G) \cong P_e(Q_{4n})$ then $G \cong Q_{4n}$.*

*Proof.* If $P_e(G) \cong P_e(Q_{4n})$, then it follows that $Pow(G) \cong Pow(Q_{4n})$ and the set $S$ of vertices in the power graph $P(G)$ that are adjacent to all other vertices satisfies $|S| > 1$. By applying Theorem 2.3, we conclude that $G \cong Q_{4n}$. □

### 3.2 Orders of groups uniquely determined by their enhanced power graphs

In this section, we focus on the order of groups and consider the following question that was also studied, [15].

> **Question:** For which $n \in \mathbb{N}$ does the condition that all pairs of groups $G$ and $H$ of order $|G| = |H| = n$, having the same number of elements of each order, imply that $G \cong H$?

The set of all such integers was denoted by $S$ in [15], where a characterization was provided for those elements of $S$ that are odd and square-free. Inspired by this, we posed a related question:

> **Question:** For which $n \in \mathbb{N}$ does the condition $P_e(G) \cong P_e(H)$, where both graphs have $n$ vertices, imply that $G \cong H$?

We denote the set of all such integers by $\bar{S}$, defined as:

$$\bar{S} = \{n \in \mathbb{N} \mid P_e(G) \cong P_e(H) \Rightarrow\ G \cong H, \text{ for all groups G and H of order n } \}$$

Since isomorphic enhanced power graphs imply that the corresponding groups have the same number of elements of each order, it follows that $S \subseteq \bar{S}$.

**Theorem 3.6.** *If $p$ is a prime number, then $2p^2 \in \bar{S}$.*

*Proof.* In [15], Lemma 1, it is shown that if $p$ and $q$ are primes such that $q \mid (p-1)$, then $p^2q \in S$ if and only if $q = 2$. Since $S \subseteq \bar{S}$, the result follows immediately.

Note that for $p = 2$, there are three abelian groups of order 8, along with two nonabelian groups, namely $D_8$ and $Q_8$. Each of these five groups has a distinct distribution of element orders. Consequently, the structure of the enhanced power graph uniquely determines the groups of order 8. □

As established in [7], the following lemma was proven for 2-groups.

**Lemma 3.7.** *( [7]) Let G be a 2-group and A be an elementary abelian 2-group. Two vertices $(a,x)$ and $(b,y)$ of the graph $Pow(G \times A)$ are adjacent if and only if one of the following conditions holds:*

1. *$x = y = 1$ and b is a power of a;*
2. *$x = y = 1$ and b is an odd power of a;*
3. *$x = 1, y \neq 1$ and b is an even power of a;*
4. *$x \neq 1, y = 1$ and a is an even power of b.*

**Remark 3.8.** *It was shown in [1] that for any finite group G, its power graph and enhanced power graph are equal if and only if every cyclic subgroup of G has prime power order. Therefore, for any finite p-group G, the enhanced power graph and power graph are equal and Lemma 3.7 extends naturally to enhanced power graphs.*

**Lemma 3.9.** *If $n \notin \bar{S}$ and $gcd(n,k) = 1$, then $nk \notin \bar{S}$.*

*Proof.* In [7], it is proved that if $G$ and $G'$ are non-isomorphic groups of order $n$ such that $\mathrm{Pow}(G) \cong \mathrm{Pow}(G')$, and $H$ is a group of order $k$ with $\gcd(n,k) = 1$, then $\mathrm{Pow}(G \times H) \cong \mathrm{Pow}(G' \times H)$. By Theorem 2.2, we can conclude that if $P_e(G) \cong P_e(G')$, then $P_e(G \times H) \cong P_e(G' \times H)$. Note that the groups $G \times H$ and $G' \times H$ are non-isomorphic. This completes the proof. □

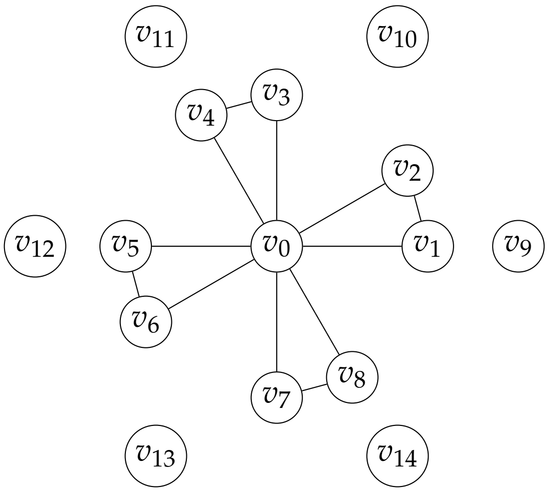


Figure 1. Enhanced power graph of $G = K_2 \times \mathbb{Z}_4$ (SmallGroup(16,10)) and $H = \mathrm{D}_8 * \mathbb{Z}_4$ (SmallGroup(16,13)), where $G$ is the direct product of the Klein four-group with a cyclic group of order 4, and $H$ is their central product. The identity vertex is removed.

**Theorem 3.10.** *Let $n = 2^{\alpha_0} p_1^{\alpha_1} \cdots p_r^{\alpha_r}$ with $r \geq 0$. If $\alpha_0 \geq 4$ or there exists $i \geq 1$ such that $\alpha_i \geq 3$, then $n \notin \bar{S}$.*

*Proof.* We first consider the case $\alpha_0 \geq 4$. Let $n = 2^4 = 16$. According to GAP's `SmallGroup` library, the group with ID 10 among the groups of order 16 is isomorphic to $G = K_2 \times \mathbb{Z}_4$, the direct product of the Klein four-group and a cyclic group of order 4. The group with ID 13 corresponds to $H$, the central product of the dihedral group of order 8 with a cyclic group of order 4.

The enhanced power graphs $P_e(G)$ and $P_e(H)$ are isomorphic; these graphs are illustrated in Figure 1. However, since $G \ncong H$, it follows that $n \notin \bar{S}$.

Now assume that $n = 2^{\alpha_0}$ with $\alpha_0 > 4$, and let $A$ be an elementary abelian 2-group of order $2^{\alpha_0-4}$. By Remark 3.8, the enhanced power graphs $P_e(G \times A)$ and $P_e(H \times A)$ are isomorphic, while the groups $G \times A$ and $H \times A$ are not isomorphic. Therefore, $n \notin \bar{S}$.

Next, suppose $n$ is not a power of 2, and write $n = 2^{\alpha_0}k$, where $k$ is an odd integer and $\alpha_0 > 4$. Since $2^{\alpha_0} \notin \bar{S}$ by the above argument and $\gcd(2^{\alpha_0}, k) = 1$, it follows from Lemma 3.9, that $n \notin \bar{S}$.

Finally, consider the case where there exists $i \geq 1$ such that $\alpha_i \geq 3$. Let $G_1 = (\mathbb{Z}_{p_i})^{\alpha_i}$ and $G_2 = \mathbb{Z}_{p_i} \times (\mathbb{Z}_{p_i})^{\alpha_i - 1}$, two non-isomorphic groups of order $p_i^{\alpha_i}$ with exponent $p_i$. Let $G'$ be any group of order $n/p_i^{\alpha_i}$. Define a bijection $\phi : G_1 \to G_2$ by mapping the standard basis of $G_1 = (\mathbb{Z}_{p_i})^{\alpha_i}$ to the generators of $G_2 = \mathbb{Z}_{p_i} \times (\mathbb{Z}_{p_i})^{\alpha_i-1}$, and extend linearly (as vector spaces over $\mathbb{F}_{p_i}$). Define $\psi : G_1 \times G' \to G_2 \times G'$ by $\psi(x,a) = (\phi(x), a)$. For elements $(x,a),(y,b) \in G_1 \times G'$, they are adjacent in $P_e(G_1 \times G')$ if $\langle (x,a),(y,b) \rangle$ is cyclic. Since $\langle (x,a),(y,b) \rangle = \langle (x,a) \rangle \langle (y,b) \rangle$, this subgroup is cyclic if $\langle x, y \rangle \leq G_1$ is cyclic (i.e., $y = x^k$ for some $k$, since $G_1$ is elementary abelian) and $\langle a, b \rangle \leq G'$ is cyclic, with orders allowing a cyclic product. In $G_2$, $\langle \phi(x), \phi(y) \rangle$ is cyclic if $\phi(y) = \phi(x)^k$, which holds since $\phi$ is an isomorphism of vector spaces preserving linear dependence. Thus, $\langle \psi(x,a), \psi(y,b) \rangle = \langle (\phi(x),a),(\phi(y),b) \rangle$ is cyclic if and only if $\langle (x,a),(y,b) \rangle$ is cyclic, because $\phi$ preserves the cyclic nature of subgroups in $G_1$ and $G_2$. Hence, $\psi$ induces an isomorphism $P_e(G_1 \times G') \cong P_e(G_2 \times G')$. Since $G_1 \ncong G_2$, we have $G_1 \times G' \ncong G_2 \times G'$, so $n \notin \bar{S}$. □

**Corollary 3.11.** *Every odd element of $\bar{S}$ is cube-free.*

### 3.3 Enhanced power graphs of nilpotent groups and groups having a normal hall subgroup

In this section, we apply Theorems 2.1 and 2.2 to derive new results concerning the enhanced power graphs of nilpotent and solvable groups.

**Theorem 3.12.** *( Theorem 4, [7]) If G and H have the same numbers of elements of each order and H is nilpotent, then also G is nilpotent.*

**Corollary 3.13.** *If $P_e(G) \cong P_e(H)$ and H is nilpotent, then also G is nilpotent.*

*Proof.* Since $P_e(G) \cong P_e(H)$, it follows that $G$ and $H$ have the same number of elements of each order. Moreover, by Theorem 3.12, this implies that $G$ is nilpotent.

□

A subgroup of a finite group is said to be a Hall subgroup if its order and index are relatively prime.

**Theorem 3.14.** *( Theorem 5, [7]) Let G and H be groups with the same number of elements of each order. If H has a normal Hall subgroup of order m and G is solvable, then also G has a normal Hall subgroup of order m.*

**Corollary 3.15.** *If $P_e(G) \cong P_e(H)$, H has a normal Hall subgroup of order m, and G is solvable, then also G has a normal Hall subgroup of order m.*

*Proof.* From $P_e(G) \cong P_e(H)$, it follows that $G$ and $H$ have the same number of elements of each order. By assumption, $H$ has a normal Hall subgroup of order $m$ and $G$ is solvable. Furthermore, by Theorem 3.14, this implies that $G$ also possesses a normal Hall subgroup of order $m$.

□

## 4 Conclusion

In this paper, we examined the extent to which the enhanced power graph of a finite group determines the structure of the group itself. By analyzing groups that share the same number of elements of each order and exploring structural properties preserved under enhanced power graph isomorphisms, we identified several important situations in which the enhanced power graph uniquely determines a group up to isomorphism. In particular, we proved that if two finite groups have isomorphic enhanced power graphs and one of them is nilpotent or possesses a normal Hall subgroup, then the other must necessarily share that property. These results provide new insight into the strength of the enhanced power graph as a group invariant and contribute to the broader program of understanding which families of groups are uniquely characterized by their associated graphs. Our work also establishes foundational criteria for determining the natural numbers $n$ for which every group of order $n$ is uniquely determined by its enhanced power graph, opening further avenues for classification within this framework. Future research may extend these ideas to infinite groups, other algebraic structures, or deeper graph-theoretic invariants associated with power-type constructions.

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