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Research Paper

Stopping sets of codes from complete bipartite graph

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Abstract. Let C be a code with parity-check matrix H. A stopping set S of size $l \leq n$ for H is an l-columns submatrix of H_s of H which does not contain a row with weight one. In this paper, we consider the code which parity-check is incidence matrix of complete bipartite graph $K_{m,n}$. These codes are LDPC codes and we obtain the stopping sets for these codes.

Keywords. linear code, stopping set, complete bipartite graph.
Mathematics Subject Classification (2020): $94B05$, $94B25$, $05C50$.

1 Introduction

Throught this paper, we consider $\mathbb{F}_2$ as a field of order 2. A linear code of *length* n and *rank* k, is a linear subspace C with *dimension* k of the vector space $\mathbb{F}_2$. The *weight* of a codeword is the number of its components that are nonzero and the *distance* between two codewords is the *Hamming distance* between them, that is, the number of components in which they are different. The Hamming distance d of the linear code is the minimum weight of its nonzero codewords, or equivalently, the minimum distance between distinct codewords. A linear binary code of length n, dimension k, and distance d is called an $[n,k,d]$-code. A *generator matrix* for a $[n, k, d]$-code C is any $k \times n$ matrix G with entries in $\mathbb{F}_2$ such that the rows of G form a basis for C. If G is a generator matrix for C, then $C = \{\mathbf{x}G | \mathbf{x} \in \mathbb{F}_2^k\}$. A *parity-check matrix* for C is an

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$(n-k) \times n$ matrix H over $\mathbb{F}_2$ such that $C = \{\mathbf{c} \in \mathbb{F}_2^n : H\mathbf{c}^T = \mathbf{0}\}$.

The *dual code* of C is $C^{\perp} := \{\mathbf{y} \in \mathbb{F}_2^n : \mathbf{x} \cdot \mathbf{y} = 0, \forall \mathbf{x} \in C\}$, where if $\mathbf{x} = (x_1, \ldots, x_n)$ and $\mathbf{y} = (y_1, \ldots, y_n)$ are in $\mathbb{F}_2^n$, then $\mathbf{x} \cdot \mathbf{y} = \sum_{i=1}^{n} x_i y_i \in \mathbb{F}_2^n$. If C is a linear code with generator matrix G and parity-check matrix H, then $C^{\perp}$ is an $[n, n-k, d^{\perp}]$-code with generator matrix H and parity-check matrix G.

Sphere of radius r around a vector u, denoted by $S_r(u)$ as $S_r(u) = \{v \in V | d(u,v) \leq r\}$. In the expression above $S_r(u)$ is the set of all vectors in the space whose distance from u is less than or equal to r, the usual definition of a sphere. The *covering radiu* $Cr(C)$ of a linear code $C \subseteq \mathbb{F}_2^n$ is the minimum positive integer r such that the union of the spheres of radius r about the codewords in C covers the whole space $\mathbb{F}_2^n$.

Low-density parity check (LDPC) codes are a class of linear block codes that allow communication close to channel capacity and are decoded iteratively. LDPC codes are among the most powerful codes currently available. The sparse structure of their parity-check matrix potentially leads to excellent distance properties and enables suby optimum iterative decoding with a complexity that grows only linearly with the block length. LDPC codes, along with an iterative decoding algorithm, were introduced by Robert Gallager already in the early 1960s [4]. The method was then more or less ignored during more than 30 years. This changed only in the mid-1990s, after the advent of turbo codes, when researchers rediscovered the advantages of codes with sparse (low-density) check matrices and became interested in codes on graphs and iterative decoding. An LDPC code is, essentially, a linear block code for which there is a very *sparse* parity-check matrix H. Here, very sparse means more specifically that the weight of each row and of each column is much smaller than the matrix dimensions. The sparse check matrix H is called a *low-density parity check matrix*. The *density* of H is defined as the number of nonzero elements H divided by the total number of elements.

Let $G = (V, E)$ be a simple graph. A *subgraph* of G is a graph (V', E'), where $V' \subset V$ and $E' \subset E$. A subgraph H of G is a proper subgraph of G if $H \neq G$. Two vertices are called *adjacent* if there is an edge between them. The *degree* of a vertex v, is the number of edges with endpoint v. Suppose that V' is a nonempty subset of V. The subgraph of G whose vertex set is V' and whose edge set is the set of those edges of G that have both ends in V' is called the subgraph of G induced by V' and is denoted by $G[V']$, we say that $G[V']$ is an *induced subgraph* of G.

Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ and $E(G) = \{e_1, e_2, ..., e_m\}$. Then (vertex-edge) *incidence matrix* of G is the $n \times m$ matrix defined as follows. The rows and the columns are indexed by $V(G)$ and $E(G)$, respectively. The (i, j)-entry of incidence matrix is 0 if vertex v_i and edge e_j are not incident, and otherwise it is 1. A *complete graph* on m vertices, denoted by K_m, is the simple graph that contains exactly one edge between each pair of distinct vertices. A *bipartite graph* G is a graph whose vertex set $V(G)$ can be partitioned into $V = A \cup B$ such that every edge of G is of the form (a, b), where $a \in A$ and $b \in B$. The *girth* of a graph is defined as the length of the smallest cycle in the graph.

A convenient way to describe an LDPC code is in terms of its factor graph. This graph is also known as *Tanner graph*(TG). There are many different Tanner graphs for a code, each one corresponding in a one-to-one fashion to a check matrix H for code. This is a natural bipartite graph defined as follows. On the left side of the graph there are n vertices, called *variable nodes*, one for each codeword position. On the right side of the graph there are $n-k$ vertices, called *check nodes*, one for each parity check (row of the parity check matrix). Each codeword bit corresponds to a variable node, labeled x_i, and each parity bit corresponds to a check node, labeled c_j. Each entry h_{ij} of 1 in the parity check matrix H corresponds directly to an edge (or connection) between variable node i and check node j in its graphical representation [7].

Let C be a $[n,k,d]$-code and H be a parity-check matrix of C. A *stopping set* S of size $l \leq n$ for H is an l-columns submatrix of H_s dose not contain a weight one row.

In other word iterative edge-removal (ER) decoders for signalling over the Binary Erasure Channel (BEC) fail on stopping sets. Stopping sets are collections of variable and check nodes in the Tanner graph of an LDPC code which greatly reduce its error correcting ability. These sets cause decoding to fail when certain variable nodes are affected by errors after transmission. Stopping sets were first described in 2002 by Di et al [2], researching the average erasure probabilities of bits and blocks over the BEC. Also in [9], Velasquez et al. studied the complexity of finding the minimum stopping sets.

The size of the smallest nonempty stopping sets from a parity-check matrix of H, denoted $s(H)$, is called the *stopping distance* of H, i.e.,

$$s(H) = min\{l \geq 1 : S_l > 0\}.$$

Although the minimum distance $d(C)$ is a fixed paramater, the stopping distance depends on the chosen parity-check matrix H for C. It is easy to verify that $s(H) \leq d(C)$.

The polynomials $S(x) = \sum_{i=0}^{n} S_i x^i$, where S_i is the number of stopping sets of size i, is called the *stopping set enumerator* of a parity-check matrix H. For any parity-check matrix H of a binary linear $[n, k, d]$-block code C, it holds that the stopping set enumerator satisfies:

$$S_i = \begin{cases} 1 & \text{for } i = 0 \\ 0 & \text{for } 1 \leq i \leq s-1 \\ \binom{n}{i} & \text{for } n - d^{\perp} + 2 \leq i \leq n \end{cases}. \tag{1}$$

The second property follows from the definition of $s(H)$, and the third property follows from the fact that:

$$wt(j) \leq n - i + 1 \leq n - (n - d^{\perp} + 2) + 1 = d^{\perp} - 1.$$

Example 1.1. *Soppose that C be a* [4,3,3]*-binary code and*

$$H = \begin{bmatrix} 1\,0\,0\,1\,1\,0 \\ 1\,1\,0\,0\,0\,1 \\ 0\,1\,1\,0\,1\,0 \\ 0\,0\,1\,1\,0\,1 \end{bmatrix},$$

then the columns $S = \{2,4,5,6\}$ are a stopping set of H as the parity-check of C. Here the parity-check equation are

$$v_1 + v_4 + v_5 = 0,$$

$$v_1 + v_2 + v_6 = 0,$$

$$v_2 + v_3 + v_5 = 0,$$

$$v_3 + v_4 + v_6 = 0.$$

The Tanner graph for C with the stopping set high lighted is shown in Figure 1.

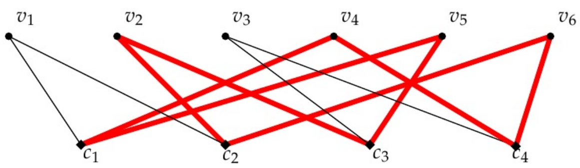


Figure 1.

Let G be a graph with H as a incidence matrix of G. Consider the code $C = C(G)$, where H is a parity-check matrix of C. These codes introduced and investigate by Dankelman et al. in [1]. They prove that the code $C(G)$ is a $[m, m - n + 1, girth(G)]$ binary linear code. These codes are LDPC code. When $G = K_m$, Esmaeili and Zaghian, found some properties of this code [3]. Also Nazari and Maimani [5] found the stopping set of this family of codes. Here we consider $K_{m,n}$, and find stopping sets of codes arrising from G.

2 The binary code $C_{m,n}$ and $C^{\perp}_{m,n}$

Suppose that $m \leq n$. Let $G = K_{m,n}$ and $H_{m,n}$ be the incidence matrix of graph G. Suppose that $C_{m,n}$ be a code with $H_{m,n}$ as the parity-check matrix. $C_{m,n}$ is a $[mn, mn - (m+n) + 1, 4]$ binary code and $C^{\perp}_{m,n}$ is a $[mn, m + n - 1, m]$ binary code [1]. These codes are LDPC code. In this section we study some parameters of this codes.

Example 2.1. For $m = 2$ and 3, we have the following matrices $H_{2,2}$ and $H_{3,3}$, respectively:

$$H_{2,2} = \begin{bmatrix} 0\,0\,1\,1 \\ 1\,1\,0\,0 \\ 0\,1\,0\,1 \\ 1\,0\,1\,0 \end{bmatrix},$$

$$H_{3,3} = \begin{bmatrix} 0\,0\,0\,0\,0\,0\,1\,1\,1 \\ 0\,0\,0\,1\,1\,1\,0\,0\,0 \\ 1\,1\,1\,0\,0\,0\,0\,0\,0 \\ 0\,0\,1\,0\,0\,1\,0\,0\,1 \\ 0\,1\,0\,0\,1\,0\,0\,1\,0 \\ 1\,0\,0\,1\,0\,0\,1\,0\,0 \end{bmatrix}.$$

We denote by $C_{m,m}$ the binary code generated by parity-check matrix $H_{m,m}$. The dual-code of $C_{m,m}^{\perp}$ is denoted by $C_{m,m}$, that is the matrix $H_{m,m}$ is a parity-check matrix for $C_{m,m}$ and a generator matrix for $C_{m,m}^{\perp}$.

It is easily followed from definition that the parity-check matrices $H_{m,m}$ are related by the recursive relation:

$$H_{m,m} = \begin{bmatrix} 1 & 0 & 0 \\ & & \\ 0_{(m-1)\times m} & H_{m-1} & \acute{I}_{m-1} \\ & & \\ I_m & 0 & 0_{(m-1)\times(m-1)} \\ & & \\ & & 1 \end{bmatrix}$$

in which $\acute{I}_{m-1}$ is the $(m-1) \times (m-1)$ binary matrix whose non-zero entries are precisely the entries located on the right-to-left diagonal.

As the first row in $H_{m,m}$ is equal to the module-2 addition of the other rows, let $H'_{m,m}$ be a matrix obtained from H by deleting the first row. Hence the first row is linear combination of other rows and the code with parity-check $H'_{m,m}$ is also equal to $C_{m,m}$. Thus the submatrix denoted by $H'_{m,m}$, is also a parity-check matrix for $C_{m,m}$. The recursive structure of $H'_{m,m}$ is given below:

$$H'_{m,m} = \begin{bmatrix} 0_{(m-1)\times m} & H_{m-1} & \acute{I}_{m-1} \\ & & \\ I_m & 0 & 0_{(m-1)\times(m-1)} \\ & & \\ & & 1 \end{bmatrix}.$$

Definition 2.2. The *rate* of a code is defined as the ratio of the number of information digits to the length.

Thus the code $C_{m,m}$ has rate:

$$R = \frac{m^2 - 2m + 1}{m^2} = 1 + \frac{-2m + 1}{m^2}$$

$lim_{m \to \infty} R(C_{m,m}) = 1.$

Example 2.3. Corresponding to $m = 2$ and $n = 2,3$. We have the following matrices $H_{2,2}$ and $H_{2,3}$, respectively:

$$H_{2,2} = \begin{bmatrix} 1\,1\,0\,0 \\ 0\,0\,1\,1 \\ 1\,0\,1\,0 \\ 0\,1\,0\,1 \end{bmatrix}, \qquad H_{2,3} = \begin{bmatrix} 1\,1\,0\,0\,1\,0 \\ 0\,0\,1\,1\,0\,1 \\ 1\,0\,1\,0\,0\,0 \\ 0\,1\,0\,1\,0\,0 \\ 0\,0\,0\,0\,1\,1 \end{bmatrix}.$$

Now for $G = K_{m,n}$, we obtain a parity-check matrix for code $C_{m,n}$ the form:

$$H'_{m,n} = \begin{bmatrix} H_{m,n-1} & I_{m\times m} \\ & \\ 0 & 0_{(n-1)\times m} \\ & \\ & 1_{1\times m} \end{bmatrix},$$

where $H_{m,n-1}$ is a parity-check matrix for code $C_{m,n-1}$.

In this paper, we find the girth of Tanner graph of $C_{m,n}$ and we find the stopping set of parity-check matrix of $H_{m,n}$.

3 Propertise of $C_{m,n}$

First we find the girth of Tanner graph of $C_{m,n}$.

Proposition 3.1. *The girth of $TG(H_{m,n})$ is 8.*

Proof. $TG(H_{m,n})$ is bipartite graph. Hence its girth is even. On the left side of the graph there are $m.n$ variable nodes (columns of the parity-check matrix) and on the right side of the graph there are $m + n$ check nodes (rows of the parity-check matrix). If $x_1 - e_1 - x_2 - e_2 - x_1$ is a cycle in $TG(H_{m,n})$ of length 4, then $e_1 = e_2 = x_1x_2$ are edges of $K_{m,n}$ which is a contradiction. Therefore $TG(H_{m,n})$ has not cycle of length 4. If $x_1 - e_1 - x_2 - e_2 - x_3 - e_3 - x_1$ is a cycle of length 6 in $TG(H_{m,n})$, then $x_1x_2x_3x_1$ is a cycle of length 3 in $K_{m,n}$, which is a contradiction. But every cycle of length 4 in $K_{m,n}$, is equivalent of cycle of length 8 in $TG(H_{m,n})$. Hence the girth of $TG(H_{m,n})$ is 8.

□

Let G be a group and X a set. A (left-) action of G on X is a map $\varphi : G \times X \longrightarrow X$ given by $(g,x) \longmapsto g \circ x$(or gx) satisfying:
$i)$ $\quad 1x = x,$
$ii)$ $\quad (gh)x = g(hx),$
for all $g,h \in G$, $x \in X$ and with 1 the identity element of G.

Definition 3.2. Let G be a group of permutations of a set X. For each $x \in X$, let $orbit(x) = \{\varphi(x) | \varphi \in G\}$. The set $orbit(x)$ is a subset of X called the orbit of x under G.

Definition 3.3. For any group G of permutations on a set X and any $\varphi \in G$, we let $fix(\varphi) = \{x \in X | \varphi(x) = x\}$. This set is called the elements fixed by φ.

Definition 3.4. Let G be a group of permutations of a set X. For each $x \in X$, let $G_x = \{\varphi \in G | \varphi(x) = x\}$. We call G_x the stabilizer of x in G. The set G_x is a subset of G and is also a subgroup of G. We know it is non-empty because the identity element will certainly fix $x \in X$.

Theorem 3.5. *[3]. Let G be a finite group of permutations of a set X. Then for any x from X,*

$$|G| = |orbit(x)||G_x|.$$

Theorem 3.6. *[8]. If G is a finite group of permutations on a set X, then the number of orbits of G on X is*

$$\frac{1}{|G|}\sum_{\varphi \in G} |fix(\varphi)|.$$

Let G be a finite group that acts on a set X. For each $g \in G$, let X^g denote the set of elements in X that are fixed by g, i.e. $X^g = \{x \in X | g.x = x\}$. Burnside's lemma asserts the following formula for the number of orbits, denoted $|X/G|$:

$$|X/G| = \frac{1}{|G|}\sum_{g \in G} |X^g|.$$

Thus the number of orbits (a natural number or $+\infty$) is equal to the average number of points fixed by an element of G (which is also a natural number or infinity). If G is infinite, the division by $|G|$ may not be well-defined; in this case the following statement in cardinal arithmetic holds:

$$|G||X/G| = \sum_{g \in G} |X^g|.$$

Burnside's Lemma can be described as finding the number of distinct orbits by taking the average size of the fixed sets.

Lemma 3.7. *Let G be a connected spanning subgraph of $K_{m,n}$. If $f \in Aut(G)$, then $f \in Aut(K_{m,n})$.*

Proof. Assume that $K_{m,n}$ is a bipartite graph, with vertex bipartition V_1 and V_2, where $|V_1| = m$ and $|V_2| = n$. Then spanning subgraph G of $K_{m,n}$ is a bipartite graph with vertex bipartition V_1 and V_2. Now if $f \in Aut(G)$, then:

$$\left.\begin{array}{ll} i) & f(V_2) = V_2, f(V_1) = V_1 \\ ii) & f(V_2) = V_1, f(V_1) = V_2 \end{array}\right\} \text{or.}$$

In (i), f induces a permutation from V_1. In the same way f induces a permutation from V_2. Then $f \in S_m \times S_n$. Hence $f \in Aut(K_{m,n})$.

In (ii), let $|V_1| = |V_2|$. Then

$$f\Big|_{V_1} : V_1 \longrightarrow V_2,$$

$$f\Big|_{V_2} : V_2 \longrightarrow V_1,$$

and so $f \in S_m \times S_n \wr S_2$, then $f \in Aut(K_{m,n})$.

□

Corollary 3.8. Number of $K_{m',n'}$ in $K_{m,n}$ is equal to

$$\begin{cases} \binom{m}{m'}\binom{n}{n'} + \binom{m}{n'}\binom{n}{m'}, & \text{if } m' \neq n', \\ \binom{m}{m'}\binom{n}{n'}, & \text{if } m' = n'. \end{cases}$$

Example 3.9. Number of $K_{1,2}$ in $K_{3,3}$ is $\binom{3}{1}\binom{3}{2} + \binom{3}{2}\binom{3}{1} = 18$.

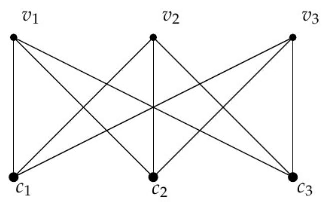


Figure 2. The graph $K_{3,3}$.

All subgraphs $K_{1,2}$ in $K_{3,3}$ are depicted in Figure 3.

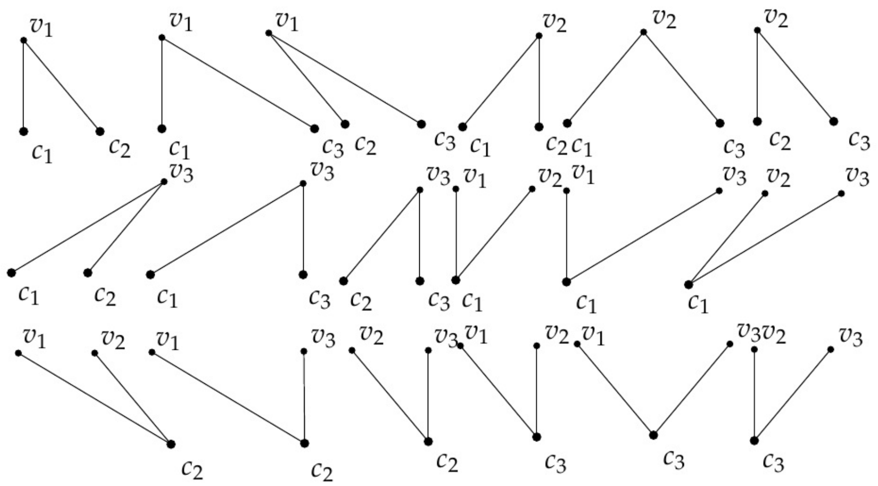


Figure 3.

Note 3.10. *Let K be a spanning subgraph of complete bipartite graph $K_{m,n}$ and $X = \{H | H \leq K_{m,n}\}$. Then $(Aut(K_{m,n})|X)$ and if $G = Aut(K_{m,n})$, then $G_K = Aut(K)$. Hence $orbit(K) = \{H | H \leq G, H \sim K\}$. So we have:*

$$|orbit(K)| = \frac{|Aut(K_{m,n})|}{|Aut(K)|}.$$

Now let K be a subgraph of $K_{m,n}$. Then $m' \leq m$ and $n' \leq n$, there exist that K is a subgraph of $K_{m,n}$. So if $T = \{H | H \leq K_{m,n}, H \sim K\}$, then:

$$|T| = \frac{|Aut(K_{m',n'})|}{|Aut(K)|} \times (number\ of K_{m',n'} in K_{m,n}).$$

Example 3.11. Know C_6 is a spanning subgraph of $K_{3,3}$. Then $Aut(C_6) = D_6$ (dihedral group of order 12) and $|Aut(C_6)| = 12$. Now if $m' = 3, n' = 3$, then

$$|T| = \frac{|Aut(K_{3,3})|}{|Aut(C_6)|} \times (\text{number of } K_{3,3} \text{in } K_{3,3}) = \frac{3!3!2}{12} = 6.$$

Example 3.12. C_4 is a subgraph of $K_{2,2}$. Since number $K_{2,2}$ in $K_{3,3}$ equal to $\binom{3}{2}\binom{2}{3} = 9$, then number C_4 in $K_{3,3}$ equal to

$$\frac{|Aut(K_{2,2})|}{|Aut(C_4)|} \times (\text{number of } K_{2,2} \text{in} K_{3,3}) = \frac{2.2.2}{2.2.2} \times 9 = 9.$$

Definition 3.13. A stopping set in a graph is a set of message nodes such that the graph induced by these message nodes has the property that no check node has degree one. The number of message nodes in the stopping set is called its size.

Theorem 3.14. *Let $X \subseteq [l]$ and H_X be a columne of H which index by X. Then H_X is a stopping set of H if and only if H_X is incidence matrix of spanning subgraph induced by X, where each vertex of X has degree not equal to* 1.

Proof. Let $X \subseteq \{1,2,...,l\}$, and $\langle X \rangle$ is denoted the spanning subgraph of G with edge set X. Then the incidence matrix of $\langle X \rangle$ is H_X. If H_X is a stopping set, then each row of H_X has weigth not equal to 1. Hence this is a subgraph of G with the degree of each vertex not equal to 1. □

Theorem 3.15. *A set of stopping set is equal to the spanning subgraph K of $K_{m,n}$. The number of K in the $K_{m,n}$ is equal to $|orbit(K)|$, K is a stopping set of rank l and we have*

$$S_l = \sum |T| = \sum \frac{|Aut(K_{m',n'})|}{|Aut(K)|} \times (number\ of\ \ K_{m',n'} in K_{m,n}).$$

Proof. By applying Note 3.10. □

Corollary 3.16. If $C = C_{m,n}$, then $S_3 = S_5 = 0$.

Proof. Every stopping set of $H_{m,n}$ of size 3, is equal to the spanning subgraph K of $K_{m,n}$ that has 3 edges. So K is a equivalent to cycle 3 in $K_{m,n}$. But the length of the smallest cycle in $K_{m,n}$ is necessarily even, and so the smallest possible cycle is 4. Hence $S_3 = 0$. □

Corollary 3.17. Let $C = C_{m,n}$. Then $S_4 = \dfrac{m!n!}{(m-2)!(n-2)!4}$.

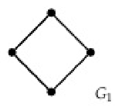
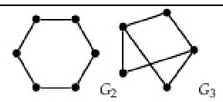
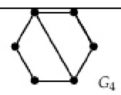
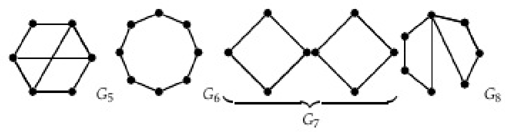
Proof. Every stopping set of $H_{m,n}$ of size 4 is equivalent to a cycle of size 4 in $K_{m,n}$. □

Remark 3.18. In Table 1, we compute the stopping set of size l, for $4 \leq l \leq 8$.

Example 3.19. If $m = 3$ and $n = 4$, then the only induced subgraph of $K_{m,n}$ of size 4, such that every vertex has degree≥ 2, is $G_1 = C_4$ and $|Aut(C_4)| = 8$ and we have $S_4 = \frac{3!4!}{1!2!4} = 18$, the only subgraph of size 4, which are indicate. A stopping set of size 6 is C_6 and $Aut(C_6) = D_6$ (dihedral group of order 12). In addition, the subgraph of size 6 is G_3 (See Table 1) and $|Aut(G_3)| = 12$ and we have $S_6 = 36$. The subgraph of size 7 is G_4 and $|Aut(G_4)| = 4$. Hence $S_7 = 72$. A stopping set of size 8, are G_5, G_6, G_7, G_8 and we have $S_8 = \frac{3!4!}{2} + \frac{3!4!}{2} = 144$. Therefore

$$S(x) = S_4x^4 + S_6x^6 + S_7x^7 + S_8x^8 = 18x^4 + 36x^6 + 72x^7 + 144x^8.$$

Table 1. Stopping sets.

S_l	graphs
$S_4 = \binom{m}{2}\binom{n}{2}\frac{\lvert Aut(K_{2,2})\rvert}{\lvert Aut(G_1)\rvert} = \frac{m!n!}{(m-2)!(n-2)!4}$	
$S_5 = 0$	———
$S_6 = \binom{m}{3}\binom{n}{3}\frac{\lvert Aut(K_{3,3})\rvert}{\lvert Aut(G_2)\rvert} + \binom{m}{2}\binom{n}{3}\frac{\lvert Aut(K_{2,3})\rvert}{\lvert Aut(G_3)\rvert} = \frac{m!n!}{(m-3)!(n-3)!6} + \frac{m!n!}{(m-2)!(n-3)!12}$	
$S_7 = \binom{m}{3}\binom{n}{3}\frac{\lvert Aut(K_{3,3})\rvert}{\lvert Aut(G_4)\rvert} = \frac{m!n!}{(m-3)!(n-3)!2}$	
$S_8 = \binom{m}{3}\binom{n}{3}\frac{\lvert Aut(K_{3,3})\rvert}{\lvert Aut(G_5)\rvert} + \binom{m}{4}\binom{n}{4}\frac{\lvert Aut(K_{4,4})\rvert}{\lvert Aut(G_6)\rvert} + \binom{m}{4}\binom{n}{4}\frac{\lvert Aut(K_{4,4})\rvert}{\lvert Aut(G_7)\rvert} + \binom{m}{3}\binom{n}{4}\frac{\lvert Aut(K_{3,4})\rvert}{\lvert Aut(G_8)\rvert} = \frac{m!n!}{(m-3)!(n-3)!2} + \frac{m!n!}{(m-4)!(n-4)!8} + \frac{m!n!}{(m-4)!(n-4)!256} + \frac{m!n!}{(m-3)!(n-4)!2}$	

4 Conclusion

In this paper we considered the codes arrived the incidence matrix of complete bipartite graph and obtained all stopping sets of parity check matrix of these family of LDPC codes.

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Data Availability Statement

Data is contained within the article.

Conflicts of Interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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